# THE STOKES LASER WITH DIFFERENT PUMP POWERS

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**Abstract:** In this paper we present the set of dynamical equations of the Raman laser. The semi-classical theory in three-dimentional approach is interested. The operating regime of Stokes laser is investigated in some cases of the pumping rate.

*Key words: Semi-classical theory, Stokes laser, Pumping regime.*

## 1. Introduction

The Raman laser is a recently developed laser technique for generating tunable high-quality laser beam in the near-infrared [5], [7]. It is based on stimulated antistokes scattering and Stokes scattering as illustrated in fig.1.

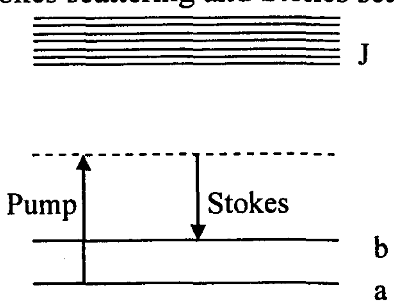


*Fig.1 Energy level diagram of the far-off-resonance Raman process*


*Fig.2 Structure of Raman laser*

Level $a$ is the ground state, level $b$ is molecular vibrational or rotational state, and the levels $J$ are the excited electronic states far-off-resonant. The transition between levels $a$ and $b$ is electric dipole forbidden. Raman scattering occurs when an incident photon (called the pump) interacts with the molecule and generates a red-shift photon (called the Stokes) and a green-shift photon (called the antistokes). Theory and experiment of Raman scattering and Raman laser (Fig.2) are investigated in many works [2], [3],[8]. Right to now, the results of Raman laser theory are limited in one-dimentional approach, only. In the work [1], Raymer et al have discussed the stimulated Raman scattering in three-dimentional approach. Generally, the Raman laser can operate at twice wavelengths, Stokes and antiStokes, depending on the reflective character of mirrors [7,9]. To enhance generated intensity of Raman laser, the pump energy should be transferred to Stokes

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field only, and then the Raman laser operates at Stokes wavelength and this laser can be called the "Stokes laser". In previous work [7], L.S.Meng has investigeted the stable regime of the Raman laser by solving a set of equations for amplitudes in one-dimentional approach, only.

In this paper, we develop a semi-classical theory for the far-off-resonance Raman laser in three-dimentional approach by providing the set of equations for power of intracavity interacting fields. The time-dependent solution to the continuous-wave Stokes laser is numerically solved and discussed.

## 2. Field amplitude equation of the stokes laser

We assume that the optical field inside the cavity will be in single spatial mode and can be written as:

$$E_q(\vec{r},t) = E_q(t)u_q(\vec{r}) \tag{1}$$

where subscript $q$ will denote the various frequency components inside the cavity (length L) such as, the pump and the Stokes. Every mode is orthogonal:

$$\iiint_{cavity} u_q(\vec{r})u_q^*(\vec{r})dxdydz = V_q \tag{2}$$

where $V_q$ is the mode volume occupied by the $q$-$th$ spatial mode.

Using the Maxwell's wave equation with slowly-varying envelope approximation, the density matrix equation with adiabatic elimination (for all time the upper levels $J$ are in steady state), we have the intracavity field equations [7]

$$\begin{aligned} &\left|\dot{E}_p(r)\right|(t)+\frac{\gamma_p}{2}\left|E_p(t)\right|(t)=-\frac{\omega_p}{\omega_s}\frac{k_p}{k_s}G(\delta)\left|E_s(t)\right|^2\left|E_p(t)\right|(t)+\frac{\gamma_{ep}}{2}\left|E_p(t)\right| \\ &\left|\dot{E}_s(t)\right|+\frac{\gamma_s}{2}\left|E_s(t)\right|=G(\delta)\left|E_p(t)\right|^2\left|E_s(t)\right| \end{aligned} \tag{3}$$

where

$$\gamma_{ep} = \frac{2c}{n_p L}\sqrt{T_{1p}}\,, \tag{4}$$

$$\gamma_{(p)s} = -\frac{c}{n_{(p)s}L}\ln\sqrt{R_{1(p)s}R_{2(p)s}} \tag{5}$$

present the intracavity energy decay rate due to primarily the loss of the mirrors (transmitance $T$, reflectance $R$), relating to external pump field, intracavity pump field and Stokes field, respectively, and

$$G(\delta) = \frac{\Phi^2}{8}c^2\varepsilon_0\alpha_g(\delta)\frac{\lambda_p}{\lambda_p+\lambda_s}\frac{\tan^{-1}(L/b_q)}{L/b_q} \tag{6}$$

is difined as a gain term, with

$$\alpha_g(\delta) = \alpha_g(0)\left(1+\frac{4\left|\Omega_{ab}\right|^2}{\gamma_{ab}\Gamma_{ab}}\right)^{-1}\left(1+\frac{\delta^2}{\gamma_{ab}^2}\frac{1}{1+4\left|\Omega_{ab}\right|^2/\gamma_{ab}\Gamma_{ab}}\right)^{-1} \tag{7}$$

is the plane-wave gain coefficient, and

$$\alpha_g(0) = \frac{2\omega_s N\hbar d_s^2\left(-D^s\right)}{n_s n_p c^2\varepsilon_0^2\gamma_{ab}} \tag{8}$$

is the line center value of the unsaturated or small-signal plane-wave gain coefficient, $b_q$ is the confocal parameter, relating to mode volume:

$$V_{p(s)} = \frac{\pi L b_q}{4k_{p(s)}},$$

$\Omega_{ab}$ is the two-phtoton Rabi frequency, $\gamma_{ab}$ is the coherence dephasing rate, $\Gamma_{ab}$ is the population decay rate and $\delta$ is the two-photon detuning for the $a \rightarrow b$ transition, $N$ is the molecule density, $d_s$ is the coupling constant to be real number, $D$ is the population difference, given as

$$D = \frac{\Gamma_{ab} D^{eq}\left(\gamma_{ab}^2 + \delta^2\right)}{\Gamma_{ab}\left(\gamma_{ab}^2 + \delta^2\right) + 4\left|\Omega_{ab}\right|^2 \gamma_{ab}} \tag{9}$$

where $D^{eq}$ is the population difference in thermal equilibrium, to be -1, i.e., all the population in ground state. In three-dimentional approach, the Fresnel number $\Phi = A/\lambda_s L$ describes the diffraction in a pencil-shape medium of cross-section $A$ and length $l$ [4].

It is necessary to write the intensity field equation in term of measurable powers, so that we can compare the experimental data with theory. In a stable two-mirror laser cavity of $L$ and in the TEM$_{00}$ spatial mode, $u_q(\vec{r})$ is of the form [10]

$$u_q(\vec{r}) = u_q(r,z) = \frac{1}{1 + i2z/b_q} e^{-r^2 k_q / b_q \left(1 + i2z/b_q\right)} \sin\left(k_q z\right) \tag{10}$$

where $r^2 = x^2 + y^2$.

From (1) and (10) one can see that $E_q(t)$ represents the peak field-amplitude of the intracavity standing-wave in space (i.e. at the center of the TEM$_{00}$ transverse profile and at the antinodes of the standing-wave along the longitudinal direction). The intracavity light intensity is calculated by

$$I_q = \frac{v_q \varepsilon_q}{2} \left|E_q(t) u_q(r)\right|^2 \tag{11}$$

where $v_q = c/n_q$ is the intracavity light speed and $\varepsilon_q = n_q^2 \varepsilon_0$ is the dielectric permeability of the intracvity medium. The intracavity power along the longitudinal direction can be calculated

$$P_q(z,t) = \int_0^\infty r dr \int_0^{2\pi} d\phi I_q = \frac{\pi \varpi_{0q}^2 n_q}{4} \sqrt{\frac{\varepsilon_0}{\mu_0}} \left|E_q(t)\right|^2 \frac{\sin^2\left(k_q z\right)}{1 + \left(2z/b_q\right)^2} \tag{12}$$

where $\varpi_{0q} = \sqrt{b_q/k_q}$ is the radius at the beam waist. If the beam inside the cavity is collimated ($z << b_q$), the peak intracavity power along the longitudinal direction is given by

$$P_q(t) = \frac{\pi \varpi_{0q}^2 n_q}{4} \sqrt{\frac{\varepsilon_0}{\mu_0}} \left|E_q(t)\right|^2 = \frac{\pi b_q}{4\omega_q \mu_0} \left|E_q(t)\right|^2 \tag{13}$$

A standing wave consists of two counter-propagating equal-amplitude traveling waves and the peak-amplitude ratio between the standing and traveling waves is 2:1 [11]. Therefore the optical power of the traveling-waves is equal to $(1/4)P_q(t)$.

Using the relation Eq.(13), one can convert Eq. (3) into

$$\dot{P}_p(t)+\gamma_p P_p(t) = -\frac{8\omega_p\mu_0}{\pi b}\frac{k_p}{k_s}G(\delta)P_s(t)P_p(t)+\gamma_{ep}\sqrt{P_p(t)P_{ep}(t)}$$
$$\dot{P}_s(t)+\gamma_s P_s(t) = \frac{8\omega_p\mu_0}{\pi b}G(\delta)P_s(t)P_p(t) \quad (14)$$

From steady-state solution, one can find the threshold pump power, given by

$$P_{ep,th} = \frac{\left(\ln\sqrt{R_{1p}R_{2p}}\right)^2}{4T_{1p}}\frac{\pi b\gamma_s}{8\omega_p\mu_0 G(\delta)} \quad (15)$$

Note that the $P_p(t)$ and $P_s(t)$ are the peak intracavity power along the longitudinal direction. Because the cavity circulating power is equal to $(1/4)P_q(t)$, the time-everage power of Stokes measured just outside the cavity mirrors takes the following form [11]:

$$P_{sout} = \frac{1}{4}T_{2s}P_s \quad (16)$$

**3. Infuence of pump rate**

For the time-dependent solution two the cw Stokes laser, we choose to numerically solve the intracavity equations (14). We numerically solve Eqs. (14) for the vibrational transition of 796→1180 nm ($\omega_p = 2\pi c/790\times10^{-9}$, $\omega_s = 2\pi c/1180\times10^{-9}$) [9], for example. Therefore the gain coefficient $\alpha_g = 1.5\times10^{-7}m/W$, the coherence decay for vibrational $\gamma_{ab}/2\pi = 250MHz$, the two-photon detuning $\delta/2\pi \approx \pm0.5GHz$, the molecular density $N = 2.4\times10^{26}m^{-3}$, the coupling constant $d_s \approx 6.0\times10^{-8}m^2Hz/V^2$, $n_s \approx n_p \approx 1$, $D^{eq} \approx 1$, the population decay $\Gamma_{ab}/2\pi \approx 10kHz$ are chosen for molecules of the Raman medium (for example $H_2$ at *25°C* and *10 atm* [6]). The reflectance of mirror $R_{1(2)s} = 0.9999$, $R_{1p} = 0.9$, $R_{2p} = 0.9999$, the length $L = 0.1m$, the curvature radius $r = 0.5m$, $b_s = \sqrt{L(2r-L)} \approx 0.3m$, and the cross section of medium $A = 12.3\times10^{-4}m^2$ and $l = 0.04m$ are chosen for the laser cavity. The external pump power are chosen to be 2, 4 and 8 times the threshold, which is calculated from (15).

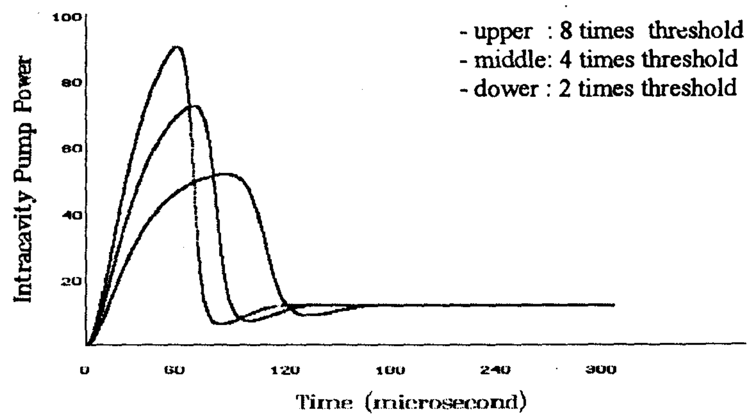


*Fig.3 The intracavity Stokes power is ploted as function of time*

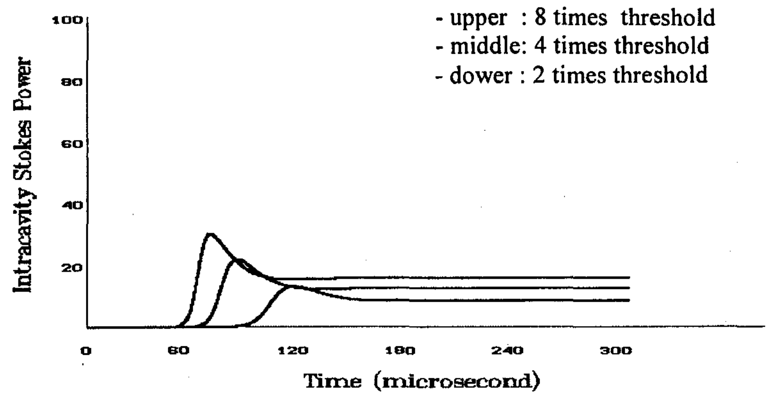


*Fig.4 The intracavity Stokes power is ploted as function of time*

The time-dependent results is given in Fig.3 for intracavity pump power and Fig.4 for intracavity Stokes one. It can be seen that at low pumping rate less than two times the threshold, the laser turns on smoothly and slowly, whereas at more then four times the threshold, there is a large overshoot.

## Conclusion

The semi-classical theory of the Stokes laser in three-dimentional approach is deriven. The intracavity intensity equations are introduced. The time-dependent solution to the cw laser is numerically culculated. Three cases of different pumping rates are calculated and ploted. Basing on this result, the influence of the cavity structure, the Raman medium, and the pumping pulse on the properties of the Stokes and Antistokes lasers will be investigated in future.

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