ON THE NONLINEAR ABSORPTION COEFFICIENT OF A STRONG ELECTROMAGNETIC WAVE BY CONFINED ELECTRONS IN DOPED SUPERLATTICES

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ABSTRACT

The nonlinear absorption coefficient of a strong Electromagnetic Wave by Confined Electrons in doped superlattices is theoretically studied by using the quantum transport equation for electrons. The dependence of absorption coefficient on the amplitude (E_0), frequency (Ω) of the external strong electromagnetic wave, doping concentration (n_D) and the temperature (T) of the system is obtained. Two limited cases for the absorption: close to the absorption threshold ($|\hbar l\Omega - \hbar\omega_0| << \bar{\varepsilon}$) and far away from the absorption threshold ($|\hbar l\Omega - \hbar\omega_0| >> \bar{\varepsilon}$) ($l = 0, \pm1, \pm2, \ldots$; ω_0 and $\bar{\varepsilon}$ are the frequency of optical phonon and the average energy of electron, respectively) are considered. The analytic expressions are numerically evaluated, plotted and discussed for a specific doping of the n-GaAs/p-GaAs superlattice. The resonant peak of the absorption coefficient appears when $\Omega = \omega_0$ and the values of the absorption coefficient are larger than they are for bulk semiconductors.

Keywords: Doped superlattices, nonlinear absorption coefficient.

INTRODUCTION

Recently, there has been considerable interest in the behavior of low dimensional systems, in particular of two dimensional systems, such as semiconductor superlattices, quantum wells and doped superlattices (DSLs). The confinement of electrons in these systems considerably enhances the electron mobility and leads to their unusual behaviours under external stimuli. As the results, the properties of low dimensional systems, especially optical properties are very different in comparison with normal semiconductors [1, 2]. Many papers have appeared dealing with the problems of optical properties in bulk semiconductors, as well as, low dimensional systems [3, 4, 5, 6]. Also, the problems of the nonlinear absorption coefficient of a strong Electromagnetic wave (Laser radiation) by free electrons in bulk semiconductors [4] and the linear absorption of a weak electromagnetic wave by confined electrons in low dimensional systems have ever been enthusiastically investigated and declared in many interesting scientific papers [3, 5, 6].

However, the nonlinear absorption problem of electromagnetic wave which has strong intensity and high frequency in DSLs still opens for studying, in this paper, we study the nonlinear absorption coefficient of a strong electromagnetic wave by confined electrons in a DSL by using the quantum transport equation for electrons. The electron - optical phonon scattering mechanism is assumed to be dominant. The nonlinear absorption coefficient is calculated by using the high frequency quantum transport equation for electrons in a DSL for two cases, which are close to the absorption threshold and far away from the absorption threshold. Then, we estimate numerical values for a specific doping of the *n-GaAs/p-GaAs* superlattice to clarify our results.

CONTENTS

1. The nonlinear absorption coefficient of a strong electromagnetic wave in DSLs.

It is well - known that the motion of an electron in a DSL is confined and its energy spectrum is quantized into discrete levels. The Hamiltonian of the electron - optical phonon system in a DSL in second quantization representation can be written as:

$$H = H_0 + U \quad (1)$$

$$H_0 = \sum_{n,\vec{k}_\perp} \varepsilon_n\left(\vec{k}_\perp - \frac{e}{\hbar c}\vec{A}(t)\right) a^+_{n,\vec{k}_\perp} a_{n,\vec{k}_\perp} + \sum_{\vec{q}} \hbar\omega_{\vec{q}} b_{\vec{q}}^+ b_{\vec{q}} \quad (2)$$

$$U = \sum_{n,n'} \sum_{\vec{q},\vec{k}_\perp} C_{\vec{q}} I_{n,n'}(q_z) a^+_{n',\vec{k}_\perp+\vec{q}_\perp} a_{n,\vec{k}_\perp}\left(b_{\vec{q}} + b^+_{-\vec{q}}\right) \quad (3)$$

where n denotes quantization of the energy spectrum in the z direction ($n = 1, 2, \ldots$), (n, $\vec{k}_\perp$) and (n', $\vec{k}_\perp + \vec{q}_\perp$) are electron states before and after scattering, $\vec{k}_\perp$ ($\vec{q}_\perp$) - the in plane (x, y) wave vector of electron (phonon), $a^+_{n,\vec{k}_\perp}$ and $a_{n,\vec{k}_\perp}$ ($b^+_{\vec{q}}$ and $b_{\vec{q}}$) are the creation and annihilation operators of electron (phonon) respectively,

($\vec{q} = (\vec{q}_\perp, q_z)$), $\hbar\omega_{\vec{q}}$ is the energy of optical phonon ($\omega_{\vec{q}} \approx \omega_0 = const$). The electron energy takes the simple form:

$$\varepsilon_n(\vec{k}_\perp) = \varepsilon_0\left(n + \frac{1}{2}\right) + \frac{\hbar^2 \vec{k}_\perp^2}{2m} \qquad (4)$$

where

$$\varepsilon_0 = \hbar\left(\frac{4\pi e^2 n_D}{\kappa_0 m}\right)^{\frac{1}{2}}$$

here, m and e are effective mass and charge of electron, respectively, κ_0 is the electronic constant, and $C_{\vec{q}}$ is the electron – phonon interaction potential. In the case of electron – optical phonon interaction, $C_{\vec{q}}$ is [4]:

$$|C_{\vec{q}}|^2 = \frac{2\pi e^2 \hbar\omega_0}{q^2}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0}\right) \qquad (5)$$

where χ_0 and χ_∞ is the static and the high-frequency dielectric constant, respectively, and

$$I_{n,n'}(q_z) = \sum_{j=1}^{s_0} \int_0^d e^{iq_z z} \Phi_n(z - jd) \Phi_{n'}(z - jd) dz \qquad (6)$$

here, $\Phi_n(z)$ is the eigenfunction for a single potential well, d is the period of DSLs and s_0 is the number of periods of DSLs.

In order to establish the quantum kinetic equations for electrons in DSLs, we use the general quantum equation for electrons distribution function [4] $f_{n,\vec{k}_\perp}(t) = \left\langle a^+_{n,\vec{k}_\perp} a_{n,\vec{k}_\perp} \right\rangle_t$:

$$i\hbar\frac{\partial}{\partial t} f_{n,\vec{k}_\perp}(t) = \left\langle [a^+_{n,\vec{k}_\perp} a_{n,\vec{k}_\perp}, H] \right\rangle_t \qquad (7)$$

where $\langle \psi \rangle_t$ denotes a statistical average value at the moment t: $\langle \psi \rangle_t = Tr(\hat{W}\hat{\psi})$ ($\hat{W}$ is the density matrix operator).

In this case, the condition $|\hbar l\Omega - \hbar\omega_0| << \bar{\varepsilon}$ is needed. We restrict the problem in the case of one

The carrier current density formula in DSLs is taken the form:

$$\vec{j}_\perp(t) = \frac{e\hbar}{m} \sum_{n,\vec{k}_\perp} \left(\vec{k}_\perp - \frac{e}{\hbar c}\vec{A}(t)\right) f_{n,\vec{k}_\perp}(t) \qquad (8)$$

because, the motion of electrons is confined along z direction in DSLs, we only consider the in plane (x, y) current density vector of electrons $\vec{j}_\perp(t)$.

Starting from Hamiltonian (1, 2, 3) and realizing operator algebraic calculations, we obtain the expression of $f_{n,\vec{k}_\perp}(t)$. Substituting $f_{n,\vec{k}_\perp}(t)$ into Eq. (8), then using the electron – optical phonon interaction potential $C_{\vec{q}}$ in Eq. (5) and the relation between the nonlinear absorption coefficient α of a strong EMW with the carrier current density $\vec{j}_\perp(t)$:

$$\alpha = \frac{8\pi}{c\sqrt{\chi_\infty} E_0^2} \left\langle \vec{j}_\perp(t) \vec{E}_0 \sin(\Omega t) \right\rangle_t \qquad (9)$$

we established the expression of α in DSLs. In this paper, we will consider two limited cases for the absorption: close to the absorption threshold and far away from this to find out the explicit formulae for the nonlinear absorption coefficient α.

1.1 The absorption far away from its threshold.

In this case, to occur the absorption of a strong electromagnetic wave in DSLs, the condition $|\hbar l\Omega - \hbar\omega_0| >> \bar{\varepsilon}$ must be satisfied. Here, $\bar{\varepsilon}$ is the average energy of an electron in a DSL. Finally, we have the explicit formula for the nonlinear absorption coefficient of a strong electromagnetic wave in DSLs for the case of the absorption far away from its threshold, which is written:

$$\alpha = \frac{\pi^2 e^4 n_0 k_B T}{c\sqrt{\chi_\infty}\hbar^2 m\Omega^3}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0}\right) \sum_{n,n'} |I_{n,n'}(q_z)|^2 \left[\frac{2m}{\hbar^2}\varepsilon_0(n-n') + \frac{2m}{\hbar}(\Omega - \omega_0)\right]^{\frac{1}{2}}$$

$$\times \left\{1 + \frac{3}{8}\left(\frac{eE_0}{2m\Omega^2}\right)^2 \left[\frac{2m}{\hbar^2}\varepsilon_0(n-n') + \frac{2m}{\hbar}(\Omega - \omega_0)\right]^{\frac{3}{2}}\right\} \qquad (10)$$

1.2 The absorption close to its threshold

photon absorption (l=1) and consider electron gas is non-degenerative. Using the electron distribution function (Boltzmann distribution function), we obtain:

$$\alpha = \frac{\sqrt{2}\pi e^4 (k_B T)^2 f_0}{8c\sqrt{\chi_\infty} m^{\frac{1}{2}} \hbar^3 \Omega^3}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0}\right)\sum_{n,n'} |I_{n,n'}(q_z)|^2 \exp\left(-\frac{\varepsilon_0(n+1/2)+\xi/2}{k_B T} - 2\sqrt{\rho\sigma}\right)$$

$$\times\left(\frac{\rho}{|\xi|\sigma}\right)^{\frac{1}{2}}\left\{1+\frac{3}{16\sqrt{\rho\sigma}}+\frac{3e^2E_0^2}{32m^2\Omega^4}\left(\frac{\rho}{\sigma}\right)^{\frac{1}{2}}\left[1+\frac{1}{\sqrt{\rho\sigma}}+\frac{1}{16\rho\sigma}\right]\right\} \qquad (11)$$

where

$$\xi = \varepsilon_0(n'-n) + \hbar(\omega_0 - \Omega)$$

$$\rho = \frac{m\xi^2}{2\hbar^2 k_B T} \; ; \quad \sigma = \frac{\hbar^2}{8mk_B T}$$

2 Numerical results and discussions

In order to clarify the mechanism for the nonlinear absorption coefficient of a strong electromagnetic wave in a DSL, in this section, we numerically evaluate, plot and discuss the expression of the nonlinear absorption coefficient for a compensated n - p *n-GaAs/p-GaAs* DSL in the case of $n=0$, $n'=1$. The characteristic parameters of the GaAs layer of the DSL are $\chi_\infty = 10.9$, $\chi_0 = 12.9$, $m = 0.067m_0$, $d = 2d_n = 2d_p = 5.10^{-9}m$, and $\hbar\omega_0 = 36.1\ meV$, ($m_0$ is mass of free electron).

2. 1 The absorption far away from its threshold

Figures (1-3) show the nonlinear absorption coefficient of a EMW in a DSL for the case of the absorption far away from its threshold. The curve increases following E_0 rather fastly and when the temperature T of the system rises up, its absorption coefficient increases slowly . It is seen that the value of the absorption coefficient increases following n_D and

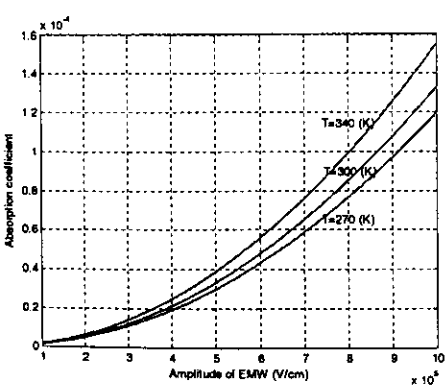

Fig. 1. The dependence of α on the amplitude E_0

diminishes very fastly following the frequency Ω. In short, the values of α in this case are very small.

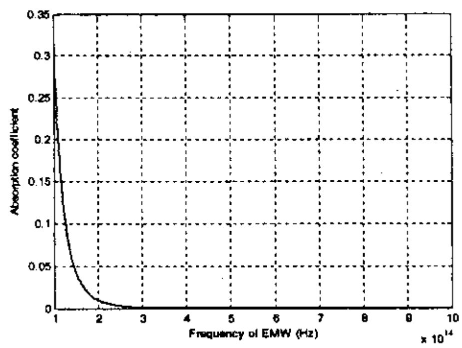

Fig. 2. The dependence of α on the frequency Ω of EMW

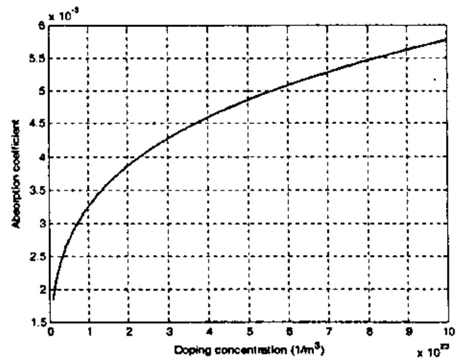

Fig . 3. The dependence of α on the doping concentration n_D

2.2 The absorption close to its threshold

In this case, the dependence of the nonlinear absorption coeficient on the another parameters are quite similar with the case of the absorption far away from its threshold. However, the values of α are much greater than above cases. Also, it is seen that α depends strongly on the electromagnetic field amplitude. That is very different from the case of the absorption close to from its threshold. Especially, figure 5 shows the absorption coefficient $\alpha(\Omega)$ as a function of Ω, the frequency of the EMW. There is a maximum coincide when $\Omega = \omega_0$ (ω_0 is the frequency of optical phonon), that means, in this case, appearing a resonant peak of the absorption coefficient.

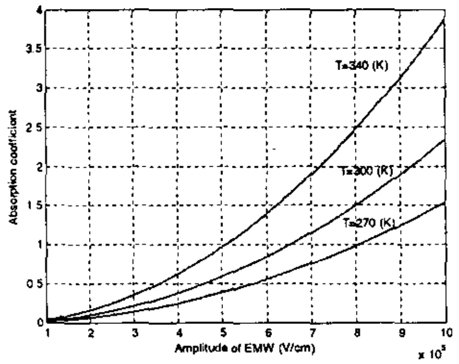

Fig. 4. The dependence of α on the amplitude E_0

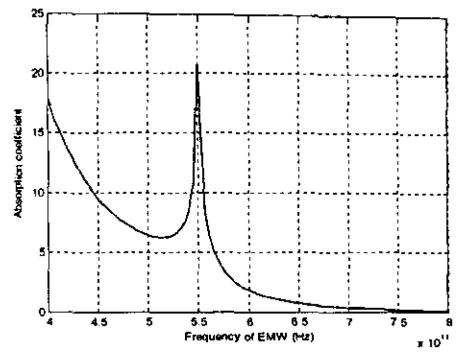

Fig. 5. The dependence of α on the frequency Ω of EMW

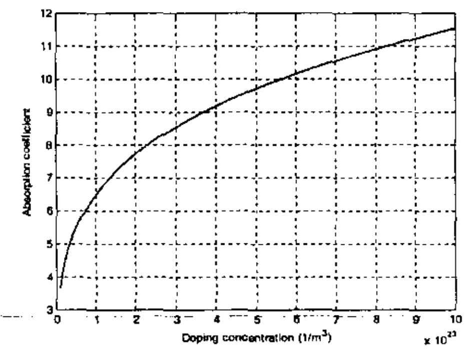

Fig.6. . The dependence of α on the doping concentration n_D

CONCLUSIONS

In this paper, we have theoretically studied the nonlinear absorption of a strong EMW by confined electrons in a DSL. We have obtained a quantum kinetic equation for electrons in DSLs. By using the tautology approximation methods, we can solve this equation to find out the expression of electrons distribution function. So that, we received the formulae of the nonlinear absorption coefficient in DSLs for two limited cases, which are far away from the absorption threshold, Eq. (10) and close to the absorption threshold, Eq. (11).

We numerically calculated and graphed the nonlinear absorption coefficient for compensated doped superlattices (*n-GaAs/p-GaAs*) to clarify the theoretical results. Numerical results present clearly the dependence of the nonlinear absorption coefficient on the amplitude (E_0), frequency (Ω) of the external strong electromagnetic wave, doping concentration (n_D) and the temperature (*T*) of the system . The resonant peak of the absorption coefficient appears when $\Omega = \omega_0$ and the values of the absorption coefficient are larger than they are for bulk semiconductors.

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References

[1]. N. Nishiguchi, *Phys. Rev. B*, **52**, (1995), 5279.
[2]. P. Zhao, *Phys. Rev. B*, **49**, (1994), 13589.
[3]. T. C. Phong, N. Q. Bau, *J. Kor. Phys. Soc.* **42** , (2003), 647.
[4]. V. V. Pavlovich and E. M. Epshtein, *Sov. Phys. Semicond.*, **11**, (1977), 809.
[5]. G. M. Shmelev, L. A. Chaikovskii and N. Q. Bau, *Sov. Phys. Semicond.*, **12**, (1978), 1932.
[6]. Nguyen Quang Bau and Tran Cong Phong, *J. Phys. Soc. Jpn*, **67**, (1998), 3875.
[7]. Herbert Bristol Dwight, *Tables of Integrals and Other Mathematical Data,* New York , 1961.
[8]. George B. Arfken and Hans J. Weber, *Mathematical methods for Physicists,* Academic Press, San Diego - NewYork - Boston - London - Sydney - Tokyo -Toronto - Dubna, 1981.