SENSITIVITY ANALYSIS OF FACTORS INFLUENCING THE RELIABILITY OF REINFORCED CONCRETE COLUMNS STRENGTHENED WITH FABRIC-REINFORCED CEMENTITIOUS MATRIX

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Abstract - This paper presents a sensitivity analysis of key factors influencing the reliability of reinforced concrete columns strengthened with Fabric-Reinforced Cementitious Matrix (FRCM) with carbon fabric. Key parameters examined include axial force, concrete compressive strength, longitudinal reinforcement diameter, and stirrup diameter. Results indicate that axial force exerts the most significant impact on column reliability, with a notable increase in failure probability as axial force rises. Enhancements in concrete compressive strength and longitudinal reinforcement diameter improve reliability by reducing failure probability. Stirrup diameter and spacing are also critical for structural safety and collapse prevention. Furthermore, the analysis reveals that while higher axial force enhances the column's lateral load-bearing capacity, it concurrently reduces the structural ductility.

Key words – Retrofit; reinforced concrete columns; carbon fabric; sensitivity analysis; reliability.

1. Introduction

Structural buildings (such as industrial buildings, bridges, and seaports) are subject to random dynamic loads under normal usage conditions. Additionally, there are random factors in the materials, dimensions, and applied loads of these structures that must also be accounted for [1]. This results in the structural response behaving stochastically, occasionally exceeding pre-determined allowable limits (damage thresholds), such as displacements or stresses that surpass permissible values. The probability of such excessive responses is termed as the structure's failure probability or its reliability probability. Thus, determining the failure probability in the presence of random input fluctuations becomes a structural reliability analysis problem [2].

It is widely recognized that model outputs in structural analysis are highly influenced by random input variables. These variables are often estimated from limited statistical data or experience, leading to inaccuracies. This raises the issue of evaluating how input variables affect model outputs to optimally adjust parameter values and enhance model accuracy. The concept of sensitivity [3], rooted in using derivatives to examine the impact of changes in related quantities, is pertinent here.

In structural mechanics, sensitivity analysis is an innovative yet highly effective method to address structural reliability challenges, as outlined above. This approach, known as structural sensitivity analysis, examines how structural response states (such as displacements, internal forces, stresses, natural frequencies, and modal shapes) depend on changes in the physical and geometric parameters (such as stiffness, density, cross-sectional area, elastic modulus, viscosity coefficient, and plate thickness) under static or dynamic loads [4].

Therefore, this study proposes a sensitivity analysis of key factors influencing structural reliability, including structural geometry, applied loads, and material corrosion. Sensitivity analysis results will reveal the most critical parameters and yield a reliability profile, indicating the relationship between the probability of structural failure and various input parameters. This has significant practical benefits, such as identifying necessary structural performance levels corresponding to design load ratings, estimating operational risks, and establishing a rational basis for maintenance decisions.

In structural buildings, reinforced concrete (RC) columns are critical structural components and may suffer from various damages, such as brittle shear failure and concrete crushing, rebar buckling, and connection issues at splice joints. To enhance the resilience of these RC columns, numerous reinforcement methods and details have been developed, including steel and concrete jackets. This study investigates the use of carbon Fabric-Reinforced Cementitious Matrix (FRCM) as the strengthened jacket for RC columns subjected to seismic loads. The study and application of new materials like FRCM for strengthening RC structures under seismic loads is also an area of global research focus. These studies primarily explore the use of innovative materials and technologies to improve earthquake resilience of existing structures, particularly in high-seismic-risk areas.

2. Retrofitting of RC columns using FRCM materials

To apply FRCM composite materials practically in civil and structural engineering for retrofitting RC concrete columns under seismic loads, a strengthening method is proposed with lap joints. Figure. 1 illustrates the schematic of the seismic strengthening of RC columns using FRCM composite materials. The specimen is reinforced with four L-shaped FRCM segments attached to the corners of the RC column, joined together with fiber mesh with a lap joint length of 200 mm. Surface coating techniques for overlapping carbon mesh are applied to improve the bonding between the meshes and the matrix.

The lap joint length of the carbon mesh and the surface coating methods described in this section were determined from previous material testing results to ensure sufficient fiber bond length within the FRCM composite material. Figure. 2 depicts the typical retrofitting procedure. Before the retrofitting procedure, the concrete surface is roughened by removing the concrete layer and sandblasting (Figure 2a). Next, to minimize stress concentration, the column corners are rounded with a radius of approximately 25 mm. After applying the first layer of mortar, woven fabrics are wrapped around and pressed so that the mortar seeps through the gaps between the fibers using a metal trowel. As shown in Figures. 2b and 2c, fabric lap joints with a length of 200 mm are created on all four sides of the specimens. The fabric is secured and kept straight in the lap joint regions using cable ties, and this process is repeated until all fabric layers are applied. The lap joint regions are impregnated with low-viscosity epoxy resin using a roller, followed by a coating of aluminum oxide powder applied with a high-speed spray gun. In the final stage, the fabric layers are fully encased in an outer layer of mortar. The total thickness of the FRCM layer is approximately 20 mm (Figure 2d).

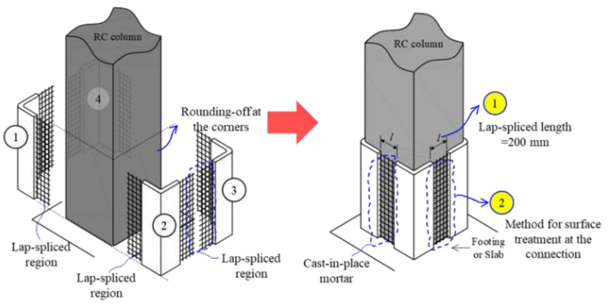
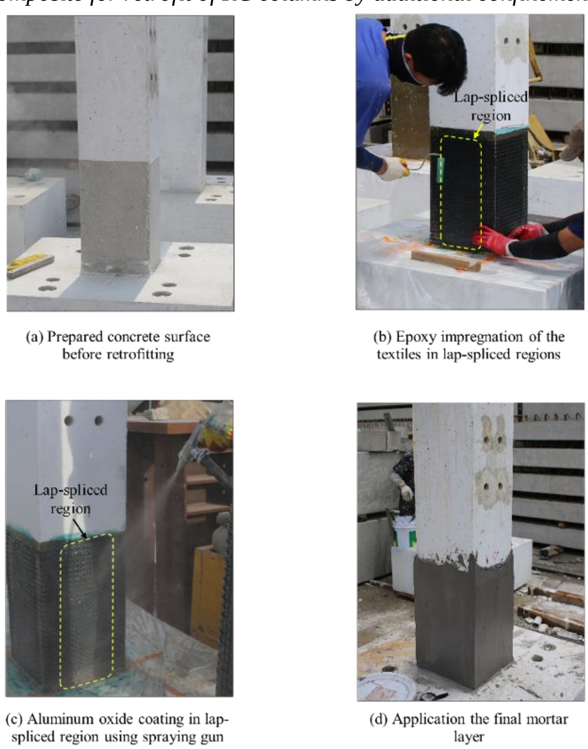


Figure 1. *Schematic detail of pre-cast segment using TRC composite for retrofit of RC columns by additional confinement*



(a) Prepared concrete surface before retrofitting

(b) Epoxy impregnation of the textiles in lap-spliced regions

(c) Aluminum oxide coating in lap-spliced region using spraying gun

(d) Application the final mortar layer

Figure 2. *Strengthening process of the column specimens*

3. Sensitivity analysis of parameters affecting the reliability of FRCM-retrofitted RC columns

3.1. Reliability analysis problem

In the structural design, many input data values are not constant; instead, they fluctuate randomly around the initial design values, typically following a specified probability distribution. These variations may arise from natural factors or human influences. This results in the fluctuation of structural responses according to a probability distribution, with some cases where the response may exceed allowable limits, such as allowable displacement or allowable stress. The probability of these response cases exceeding allowable limits is known as the structural failure probability or structural unreliability probability. Determining the failure probability of a structure under random fluctuations in input factors is referred to as structural reliability analysis [5].

The first step in calculating the reliability or failure probability of a structure is to select the safety or failure criteria of the element or structure under consideration, along with the appropriate load or strength parameters, referred to as the basic variables X_i, and their functional relationship in accordance with the applicable standard. Mathematically, the performance function for this relationship can be described by:

$$M = g(x_1, x_2, \dots, x_n) \quad (1)$$

where $x_1, x_2, \dots, x_n$ are random variables that directly influence the structural state.

The failure surface or limit state is defined when $M = 0$. This represents the boundary between the safe and unsafe regions in the parameter space and indicates the state at which a structure no longer fulfills its intended design function. The limit state equation plays a crucial role in developing reliability analysis methods.

The limit state can be an explicit or implicit function of the basic random variables and may be in a simple or complex form. Reliability analysis methods are developed corresponding to the limit states based on their characteristics and complexity levels.

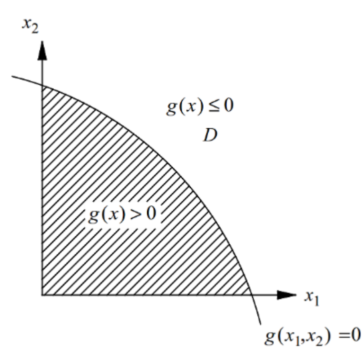


Figure 3. *The definition of Reliability with Two Random Variables*

From Eq. (1), the failure occurs when $M < 0$. Therefore, the failure probability (p_f) can be expressed as follows:

$$p_f = \int \dots \int_{g(.)<0} f_x(x_1, x_2, \dots, x_n) dx_1 dx_2 \dots dx_n \quad (2)$$

where $f_x(x_1, x_2, \dots, x_n)$ is the joint probability density function of the basic variables $X_1, X_2, \ldots, X_n$, and the integration is performed over the unsafe region, meaning that $g(.) < 0$. If the random variables are statistically independent, then the joint probability density function can be replaced by the product of the probability density functions of each variable.

3.2. Sensitivity analysis of parameters affecting the reliability of RC columns

In structural reliability analysis, sensitivity analysis [6-8] is employed to measure the extent to which input variables affect the failure of the structure [9,10]. In this study, we apply a sensitivity analysis method for reliability proposed by authors Sinan Xiao and Zhenzhou Lu [11].

In this proposed method, the output of the model is divided into two domains: the failure domain and the safe domain. The central idea is that if the conditional probability density function of the input variable concerning failure differs significantly from its unconditional probability density function, then that input variable is sensitive to the failure of the structure.

Given that $\mathbf{X} = (X_1, X_2, \ldots, X_n)$ is an n-dimensional vector of random input variables. All these input variables are independent of each other. The probability density function (PDF) of X_i is denoted as $f_{X_i}(x_i)$ for i=1,…,n, and the joint probability density function of **X** can be expressed as follows:

$$f_X(x) = \prod_{i=1}^{n} f_{X_i}(x_i) \tag{3}$$

The output variable **Y** is defined by: $\mathbf{Y} = g(X_1, X_2, \ldots, X_n)$ where $g(X_1, X_2, \ldots, X_n)$ represents the limit state.

Let $F = \{g(\mathbf{X}) \leq 0\}$ denote the failure state of the structure. Thus, the probability of failure can be defined as: $P(F) = P(g(\mathbf{X}) \leq 0)$.

In structural reliability analysis, the output of the model can be separated into two domains, F and $\bar{F}$, where F represents the failed structure $(g(\mathbf{X}) \leq 0)$ và $\bar{F}$ represents the non-failed structure (safe condition). F and $\bar{F}$ are two complementary sets. If F is determined, $\bar{F}$ is also determined.

Now, we will examine the difference between the conditional probability density function concerning failure $f_{X_i}(x_i|F)$ and the original probability density function $f_{X_i}(x_i)$ of X_i. This difference can be represented by the area between these two probability density functions, that is:

$$d_i = \int_{X_i} \left| f_{X_i}(x_i) - f_{X_i}(x_i|F) \right| dx_i \tag{4}$$

According to the previous studies, a significant magnitude of d_i (which measures the difference between the original probability density function and the conditional probability density function concerning the failure of the variable X_i) indicates that the variable X_i significantly affects the failure of the structure.

Definition of the Reliability Sensitivity Index (S_i): The reliability sensitivity index S_i is defined as half the value of d_i:

$$S_i = \frac{1}{2} d_i = \int_{X_i} \left| f_{X_i}(x_i) - f_{X_i}(x_i|F) \right| dx_i \tag{5}$$

This formula represents the sensitivity of the input variable X_i to the failure of the structure through the difference between the original probability density function and the conditional probability density function regarding failure. The larger the value of S_i, the more sensitive the variable X_i, indicating the greater influence on the failure of the structure.

The detailed properties of the index S_i are presented in Table 1.

Table 1. *Properties of the Sensitivity Index S_i*

Properties	Meaning/Condition
$0 \leq S_i \leq 1$	X_i affects F, with a sensitivity index S_i
$S_i = 0$	X_i and F are independent variables, meaning that X_i does not affect the failure of the structure

3.3. Determination of Reliability Sensitivity Index Using the Monte Carlo Method

In this section, a Monte Carlo Simulation (MCS) process with a single sample set is utilized to estimate the proposed sensitivity indices of reliability. For the individual sensitivity index S_i, the key issue is the estimation of the conditional probability density function of failure $f_{X_i}(x_i|F)$ for X_i. It is evident that when $f_{X_i}(x_i|F)$ is estimated using failure samples, S_i can be easily determined.

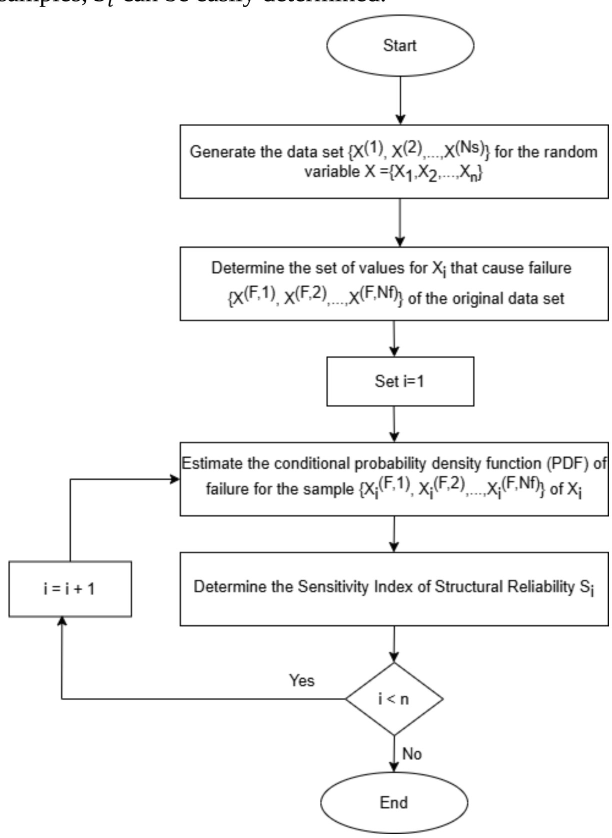


Figure 4. *Block diagram for calculating the reliability sensitivity index S_i*

4. Limit State Function for FRCM-retrofitted RC columns

The shear demand (V_p) of the RC column is calculated as the ratio of the nominal moment capacity (M_n) to the column length (L) according to Eqs. (6) and (7) [12]:

$$V_p = \frac{M_n}{L} \quad (6)$$

$$M_n = C_c\left(\frac{h}{2} - \frac{\beta_1 c}{2}\right) + \sum A_{si} f_{si}\left(\frac{h}{2} - d_i\right) \quad (7)$$

where:

C_c is the compressive force in the compression zone;

A_{si} is the area of steel reinforcement;

f_{si} is the stress of steel reinforcement;

d is the distance from reinforcement layers to the extreme compression fiber.

The shear capacity (V_n) of the RC column is calculated according to the ACI 318-14 Standard using Eqs. (8) to (9):

$$V_n = V_c + V_s \quad (8)$$

$$V_c = \frac{1}{6}\left(1 + \frac{P}{14bh}\right) bd\sqrt{f'_c} \quad (9)$$

$$V_s = \frac{A_v f_{yt} d}{s} \quad (10)$$

where:

V_c is the shear contribution of concrete,

V_s is the shear contribution of transverse reinforcement,

f'_c is the concrete compressive strength,

b is the width of the column cross-section,

d is the effective depth of the column cross-section.

The shear capacity (V_n) of the FRCM-retrofitted RC column is calculated as follows [12]:

$$V_n = V_c + V_s + V_f \quad (11)$$

where V_f is the shear contribution of the FRCM jacket, calculated using ACI 549.4R-13 (2013):

$$V_f = nA_f f_{fv} d_f \quad (12)$$

$$f_{fv} = \varepsilon_{fv} E_f \quad (13)$$

$$\varepsilon_{fv} = \varepsilon_{fu} \le 0.004 \quad (14)$$

Where:

n is the number of fabric layers, Af is the area of one FRCM layer.

d_f is the thickness of FRCM jacket,

ε_{fv} is the effective strain of FRCM,

E_f is the cracked elastic modulus of FRCM,

ε_{fu} is the ultimate strain of FRCM.

ε_{fu} and E_f in Eqs. (12) và (13) is directly obtained from the tensile tests of the FRCM composite [13].

Thus, V_p represents the shear force developed in the column to resist the lateral loading, which typically occurs due to wind, earthquakes, or other dynamic loads. This level of loading significantly affects the strength and ductility of the column, especially under cyclic loading conditions and accumulated inelastic deformations. Understanding V_p is crucial in the design of RC columns to ensure that they can withstand lateral loads without failure.

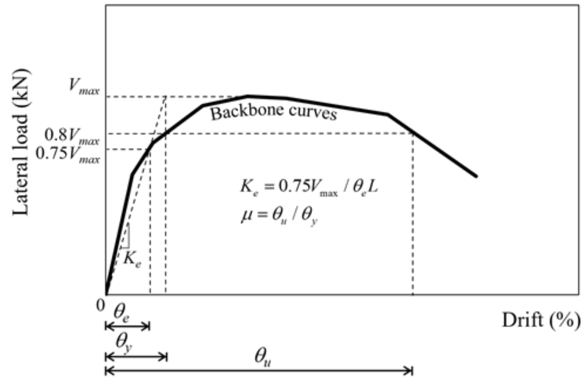


Figure 5. *Determination of the performance index from backbone curves*

The relationship between Vp and drift is a crucial factor in evaluating the performance of RC columns under lateral loading. As lateral load increases, the drift also increases, indicating the deformation of the column. The initial stiffness (Ki) is high when the drift is low, meaning that the column effectively resists deformation. However, as the drift increases, stiffness decreases, indicating that the column becomes more susceptible to deformation. Figure. 4 clearly illustrates this change, showing the initial shear capacity gradually diminishing as drift increases. After reaching the maximum load (Vmax), the load-carrying capacity of the column decreases, indicating a degradation in the capacity to resist further lateral loading. Analyzing this relationship helps identify the point at which the column can withstand before failure occurs, thereby optimizing the design and reinforcement.

Drift ratio is a significant measure for assessing the extent of deformation and the failure state of reinforced concrete columns when subjected to lateral loads. In the design and reinforcement of RC columns, three primary failure states are commonly considered: Immediate Occupancy (IO), Life Safety (LS), and Collapse Prevention (CP) [14]. The IO state represents a condition of minor deformation, where the column still retains its load-bearing capacity without significant structural damage. The LS state indicates the beginning of structural damage to the column, although it does not pose a threat to life. Finally, the CP state represents the maximum level of deformation that the column can endure before collapse. Depending on the standards, each country specifies limit drift values for the three (or potentially more) failure states of a structure.

After retrofitting using FRCM materials, the load-carrying capacity and deformation of the column are significantly improved, reducing the risk of damage in the IO and LS states while enhancing resilience in the CP state. This ensures that the column can maintain better performance under severe lateral loading conditions, protecting lives and property during earthquakes or strong wind events.

In reliability analysis, the limit state function is defined as a tool for assessing the safety of structures under various loading conditions, specifically based on the criterion $drift_{max} \geq drift_{limit}$. The limit state function (g) is expressed as:

$$g(\boldsymbol{X}) = drift_{limit} - drift_{max} \qquad (15)$$

where:

$drift_{max}$ is the maximum drift that the structure experiences under lateral loading,

$drift_{limit}$ is the limit value of drift determined based on design standards or performance requirements,

X represents the input random variables, such as loads, materials, and cross-sectional dimensions.

The determination of this limit state function allows engineers to evaluate the probability of failure of the structure and implement reinforcement measures to improve reliability, ensuring that the structure can operate safely under the anticipated loading conditions.

5. Results of the sensitivity analysis

In analyzing the probability density function (PDF) of rotational displacement (drift) for three different failure states (Immediate Occupancy - IO, Life Safety - LS, and Collapse Prevention - CP), clear distinctions among the states are observed. The IO state (represented by the blue curve) has a PDF with a prominent peak at a displacement value of approximately 0.5 to 1, indicating that the structure can withstand small displacements before affecting the elastic phase. The LS state (represented by the red curve) has a very high and narrow peak around a displacement value of 1, reflecting the greatest risk to life safety as the displacement values are primarily concentrated at this level. Meanwhile, the CP state (represented by the purple curve) shows a broader PDF with multiple smaller peaks, suggesting that the structure can withstand various displacement levels before experiencing a total collapse risk.

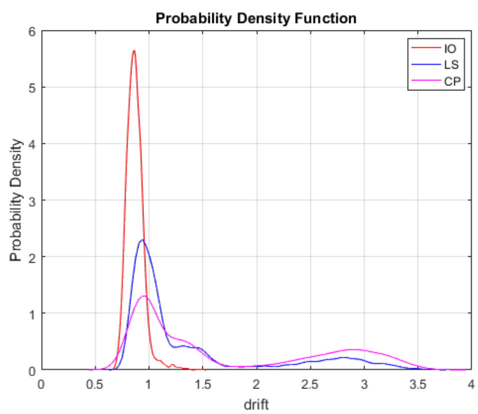


Figure 6. *The probability density function of rotational displacement*

When analyzing the PDF of the limit state function $G(\boldsymbol{X})$ for the three aforementioned failure states, similar trends are observed. The IO state has a PDF of $G(\boldsymbol{X})$ with a small peak in the $G(\boldsymbol{X}) > 0$ region, indicating that the structure maintains its elasticity without significant damage. The LS state exhibits a very high and narrow peak around $G(\boldsymbol{X})$ close to 0, reflecting the highest risk of failure with minimal variations in $G(\boldsymbol{X})$. The CP state shows a broad PDF of $G(\boldsymbol{X})$ with multiple small peaks in the $G(\boldsymbol{X}) < 0$ region, suggesting that the structure can withstand various levels of damage before a complete collapse. These analyses provide a comprehensive view of structural behavior under different failure states, supporting the assessment of structural safety and resilience across various scenarios.

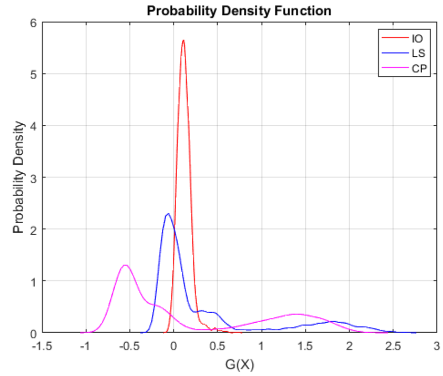


Figure 7. *The probability density function of the limit state*

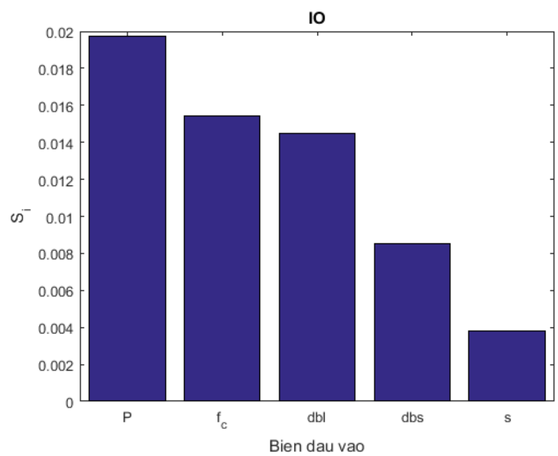


Figure 8. *Reliability sensitivity index, S_i, at the "Immediate Occupancy" state*

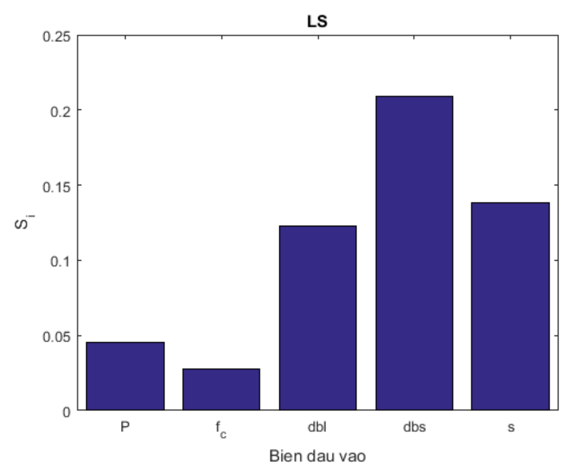


Figure 9. *Reliability sensitivity index, S_i, at the "Life Safety" state*

The sensitivity of input parameters affecting structural reliability, as indicated by the sensitivity index, S_i, reveals the influence of each parameter on structural reliability across the three different failure states (IO - Immediate

Occupancy, LS - Life Safety, CP - Collapse Prevention). We will analyze each chart in detail.

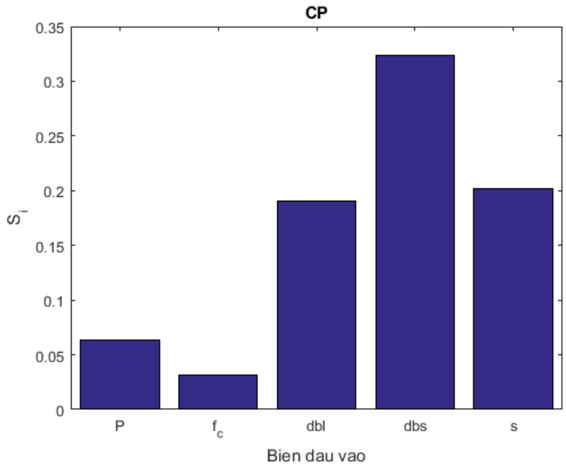


Figure 10. *Reliability sensitivity index, S_i, at the "Collapse Prevention" state*

From the Figures. 8–10, we observe that:

- Axial force (*P*) has a significant impact on reliability in the IO state, but this influence gradually decreases in the LS and CP states.
- Concrete compressive strength (f'_c) has the least impact across all three failure states.
- Longitudinal steel diameter (d_{bl}) has a moderate influence across all three states, indicating the important role of longitudinal reinforcement in the columns.
- The diameter (d_{bs}) and spacing (*s*) of stirrups have a considerable influence on both the LS and CP states, especially the stirrup diameter. This emphasizes the critical role of stirrups in ensuring life safety and preventing structural collapse.

These results can assist design and construction engineers in focusing on key parameters to enhance structural reliability across different failure states.

6. Conclusion

In this study, through sensitivity analysis of parameters affecting the reliability of RC columns retrofitted by FRCM materials, the results demonstrate that axial load is the most significant factor influencing the failure probability of the column. As the axial load increases, the failure probability also increases; however, the lateral load-carrying capacity of the column concurrently improves. This creates a balance between load-carrying capacity and ductility, necessitating careful consideration by design engineers to achieve optimal performance.

Additionally, increasing the compressive strength of concrete and the diameter of longitudinal steel bars helps to reduce the probability of failure, thereby enhancing the reliability of the columns. Notably, parameters related to the stirrups, such as diameter and spacing, play a crucial role in ensuring structural safety, especially under large dynamic load conditions such as earthquakes or strong winds. These results indicate that stirrups help prevent collapse in extreme conditions.

The research findings also provide practical insights into the design and retrofit of RC columns. Optimizing these structural parameters will help reduce the risk of damage, extend the service life of the structure, and minimize risks in dynamic loading scenarios.

Overall, the study has clarified the relationship between structural factors and the load-carrying capacity of RC columns retrofitted by FRCM materials, thereby offering practical solutions for improving the reliability and safety of construction projects.

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