

ANALYSIS OF THE DYNAMIC IMPACT FACTOR OF SONG PHAN BRIDGE UNDER RANDOM LOADS COLLECTED FROM THE DAU GIAY WEIGHING STATION AND ROAD MANAGEMENT AREA IV

Xuan Toan Nguyen, Cong Thuat Dang, Thi Kim Loan Nguyen*

The University of Danang - University of Science and Technology, Vietnam

*Corresponding author: ntkloan@dut.udn.vn

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Abstract - This paper investigates the Dynamic Impact Factor (DIF) of vertical displacement in a three-span girder bridge with a continuous thermal deck subjected to random vehicle loads. Actual vehicle load data collected from the Dau Giay weighing station and Road Management Area IV were utilized in the analysis. The study employed the non-parametric density estimation method to generate random data sets based on the probability density function of the collected vehicle loads. A total of 10000 simulations were conducted, applying the analysis to the Song Phan Bridge on the Phan Thiet - Dau Giay highway in Vietnam. The research findings indicate that the probability distribution of the random variables is highly complex. Furthermore, the Dynamic Impact Factor corresponding to the overload loads from Road Management Area IV tends to increase and exceeds the dynamic coefficients currently utilized in bridge design standards, according to AASHTO and TCVN 11823-13:2017.

Key words – Dynamic impact factor (DIF); random vehicle loads; Song Phan bridge; Dau Giay weighing station; Road Management Area IV.

1. Introduction

The dynamic response of a bridge is most commonly influenced by its span, natural frequencies, and road surface irregularities. However, several other factors related to traffic characteristics, such as vehicle speed, axle load, vehicle configuration, and dynamic characteristics, as well as bridge-specific characteristics, including bridge type, condition of major structural components, expansion joints, supports, and its damping behavior in response to loads also play an important role [1]. In reality, many effects of excitation forces on bridge structures are random, such as wind load, rain load, moving vehicle load, earthquake, etc. In which moving vehicle load is a load that acts directly, regularly, continuously, and always changes in space and time. Analyzing bridge vibrations on the interaction model between bridge and vehicle from a random perspective has been of interest to scientists in the world and in the country very early. Some researchers have solved different aspects of the problem by using a random approach of bridges under moving vehicle loads according to various computational models. Zibdeh [2] dealt with the stochastic vibration of a simple elastic beam subjected to a random load moving at a velocity varying with time. The problem was formulated based on the Euler-Bernoulli beam theory and applied a stochastic method, generating partial differential equations to describe the behavior of the system. S'niady et al. [3] presented an approach to obtain the probabilistic characteristics (deflection,

stress) of the beam response due to a load moving at a random velocity. Lu et al. [4] proposed an efficient algorithm for non-stationary stochastic vibration of a bridge-vehicle system. The pseudo-excitation method (PEM) was extended to handle the stochastic analysis of time-dependent bridge-vehicle systems, in which the statistical characteristics of the dynamic responses are calculated. Yin et al. [5] considered the non-stationary random vibration of the bridge when the vehicle speed changes. The mathematical model was established based on the finite element model, in which the Gaussian random process was presented by the Karhunen-Loève expansion, and the equations were solved by the Newmark- β method. Chang [6] applied the finite element method to perform a statistical kinematic analysis of the bridge-vehicle interaction with randomness in material properties and moving loads. Sorrentino [7] studied the analysis on a simple beam loaded by a series of concentrated forces moving in the same direction, with random occurrence, continuous random speed, and constant random amplitude. Camara et al. [8] modeled the bridge-vehicle interaction applicable to medium to long-span bridges subjected to random traffic loading. Montenegro et al. [9] described a method for evaluating the dynamic impact factor in bridges based on irregular configurations determined by probability distributions. Shao et al. [10] presented a numerical simulation method for vehicle-bridge interaction for long-span suspension bridges under random traffic loading in the linear range. The above studies have mentioned models that are closer to reality and have begun to consider random factors such as speed, position, distance of vehicles, excitation force, roughness of the bridge surface, and vehicle load according to the standard distribution law. However, the research results on the vibration of bridge structures under the influence of mobile vehicles with randomly changing loads are still very limited.

In general, this coefficient represents the relationship between the total response observed in a structure under dynamic loading and its corresponding static component. The importance of dynamic bridge loading was recognized as early as the 1930s when the British Standard Loading Code introduced a DIF of 1.5 [11]. Around the same period, the American Association of State Highway and Transportation Officials (AASHTO) [12] issued the Standard Specifications for Highway Bridges, establishing the first correlation between bridge length and the DIF. Initially, the DIF was limited to 1.25, which was later increased to 1.3 in 1989 (AASHTO) [13]. A review of the

various factors influencing highway bridges constructed by different countries can be found in GangaRao [14]. Several studies have demonstrated that the DIF calculated using mathematical models for different bridge responses varies, and some suggest that the DIF obtained from different bridge responses should be treated separately ([15-17]). Deng et al. [18] performed an extensive literature review on the use of the DIF in multiple countries. The results correspond to the bridge codes of each country, resulting in differences in DIF calculations.

Recently, developed countries have applied many advanced bridge construction technologies and materials to respond to random fluctuations in vehicle loads. These systems provide real-time data on the load as well as the state of the bridge, helping engineers detect problems early and promptly adjust treatment methods to the structure. In recent years, Vietnam's infrastructure has been invested in construction and developed rapidly. The impact of vehicle loads, especially overloaded vehicles, on the vibration and safety of bridge structures is of interest to many scientists. In fact, the load of vehicles passing over the bridge varies greatly and depends a lot on the type and volume of goods loaded on the vehicle, so this quantity is random. The collection of actual data on the load and vibration of bridges is still limited, especially in developing countries like Vietnam. Currently, most studies rely on simulation data or data from atypical conditions, which do not reflect the actual traffic and load conditions in Vietnam. The lack of real data on the impacts of overloaded vehicles on bridges reduces the ability to verify and calibrate analytical models. In addition, there are very few specific studies on random loads in real conditions in Vietnam, where overloaded vehicles often occur. This study collects real vehicle load data from the Dau Giay weighing station and Road Management Area IV, and applies it to the DIF analysis of a three-span girder bridge with a thermally continuous slab, the Song Phan bridge. At the same time, compare the results obtained when using two different sets of vehicle load data to predict the danger zone during the operation of the bridge structure subject to different mobile vehicle loads.

2. Collect and process actual load data of Dau Giay weighing station and Road Management Area IV

2.1. Collect and enrich vehicle loads algorithm

Weigh-in-motion (WIM) systems are widely used globally, from developed to developing countries. This is a system designed to measure the weight of vehicles moving on the road using sensors placed on the road surface. The sensors are designed to measure the impact force of vehicles moving on the road and convert it into weight data corresponding to those vehicles. The data collected from the WIM system can be used to calculate the average weight and maximum load of vehicles moving on the road, helping road managers assess the durability of the road and manage road loads more effectively. In this study, the WIM database was taken from the Dau Giay weighing station - Dong Nai province and Road Management Area IV. The actual load obtained from the Dau Giay weighing station is the operating load with some cases of overload. The load collected at Road Management Area IV is understood as the overloaded

vehicle load, that is, the load exceeds the allowable limit. The density estimation methods were used to estimate the probability density function (PDF) of a variable from experimental data. To perform the steps of estimating the PDF of a variable from experimental data, many empirical methods can be used. In this section, the Kernel density estimation method is used. This method involves using a kernel function to smooth the observed data points, creating a continuous approximation of the PDF. This is an efficient way to provide a clear and continuous estimate of the density function from discrete data points. Estimating the PDF, using Kernel density function estimation (KDE) is a powerful approach to estimate the PDF from a real data sample, which helps to visualize the distribution of the data without making specific assumptions about the shape of the prior distribution [19].

The block diagram to simulate experimental data using the Monte Carlo method based on the collected data from weighing stations is as follows:

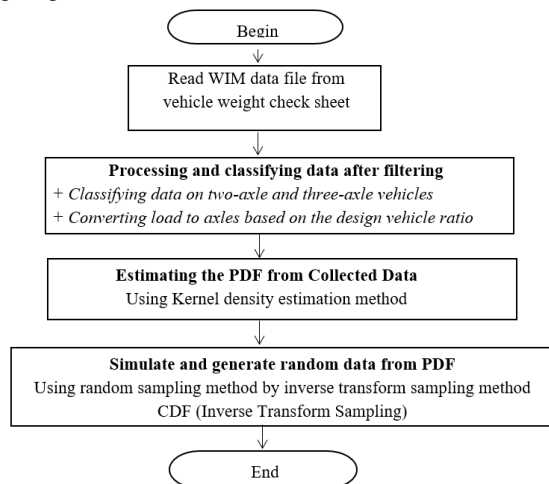


Figure 1. The block diagram for generating random data set

2.2. Dau Giay weighing station

Figure 2 presents the frequency histogram of vehicle load occurrences and the probability density function derived from the data processing collected at the Dau Giay weighing station according to Nguyen Lan [20]. Random samples from the dataset of m_{1n} values representing the load of the third axle are illustrated in Figure 3.

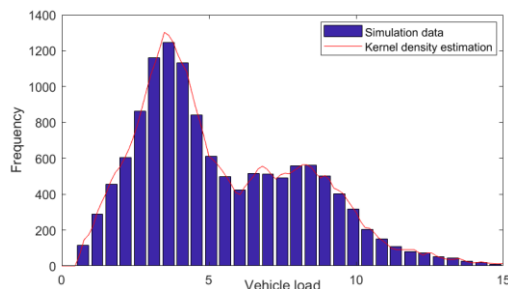


Figure 2. The frequency distribution chart and PDF

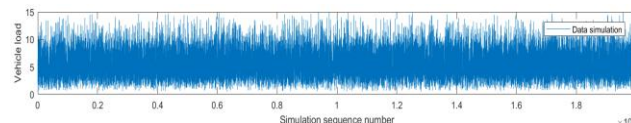


Figure 3. The randomness of the vehicle

2.3. Road Management Area IV

Road Management Area IV (RMA) is one of the road management agencies in Vietnam and is responsible for ensuring traffic safety on the country's key routes. Through the above algorithm, a random dataset of 10000 times the value of the third axle load from 2018 to 2021 was generated and shown in Figure 4. By using this random dataset, conducting simulations, and statistical analysis, the results of the random representations of the third axle load are shown in Figure 5.

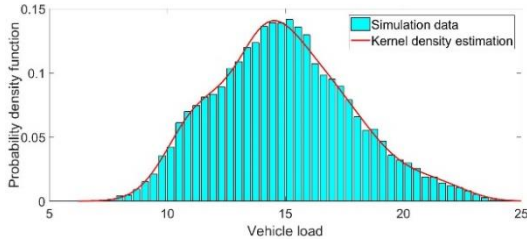


Figure 4. The input load data of RMA IV

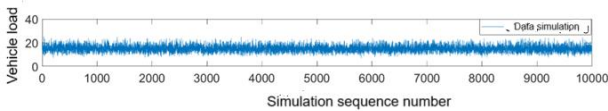


Figure 5. The randomness of the vehicle

3. Vehicle-bridge coupled system

The structure of the reinforced concrete I-girder bridge, comprising three spans and featuring a link slab, is subjected to vehicle loads, as illustrated in Figure 6.

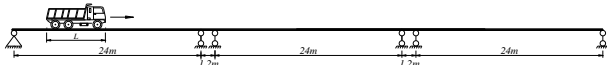


Figure 6. Schematic of a three-axle vehicle moving on the bridge

Vehicle load is modeled as a three-axle system. Let m_{1n} represent the mass of the vehicle, including the goods transported to the n^{th} axle, and m_{2n} denote the mass of the n^{th} axle. The dynamic interaction model between a three-axle vehicle and a girder element is shown in Figure 7.

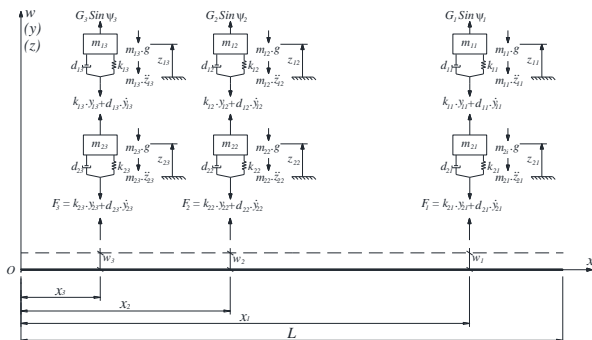


Figure 7. Vehicle-bridge dynamics interaction model

In this model, $w_n(x_n, t)$ represents the vertical displacement of the girder element at the n^{th} axle of the vehicle, while z_{1n} is the vertical displacement of the chassis at the n^{th} axle and z_{2n} denotes the vertical displacement of the n^{th} axle itself. The term y_{1n} refers to the relative displacement between the chassis and the n^{th} axle, whereas y_{2n} indicates the relative displacement between the n^{th} axle and the girder element. The engine excitation force at the

n^{th} axle, modeled as a harmonic function, is expressed by $G_n \cdot \sin \psi_n$. The suspension system at the n^{th} axle is characterized by a spring constant k_{1n} and a damping coefficient d_{1n} , while the tire system is represented by the spring constant k_{2n} and the damping coefficient d_{2n} . Here, L is the length of the girder element and x_n is the coordinate of the n^{th} axle of the vehicle at time for $n = 1, 2, 3$.

As outlined in [21], the differential equations governing the behavior of a girder element under a distributed load $p(x, z, t)$, while accounting for both internal and external friction effects, are expressed as follows:

$$EJ_d \cdot \left(\frac{\partial^4 w}{\partial x^4} + \theta \cdot \frac{\partial^5 w}{\partial x^4 \cdot \partial t} \right) + \rho F_d \cdot \frac{\partial^2 w}{\partial t^2} + \beta \cdot \frac{\partial w}{\partial t} = p(x, z, t) \quad (1)$$

In this context, EJ_d represents the bending stiffness of the girder element, while ρF_d denotes the mass per unit length of the girder. The coefficients θ and β account for the internal and external friction of the girder element, respectively. The term $p(x, z, t)$ refers to the uniform load applied horizontally across the girder element.

By applying d'Alembert's principle, the dynamic equilibrium of each mass m_{1n} , m_{2n} along the vertical axis can be expressed as follows:

$$m_{1n} \cdot \ddot{z}_{1n} + d_{1n} \cdot \dot{z}_{1n} + k_{1n} \cdot z_{1n} - d_{1n} \cdot \dot{z}_{2n} - k_{1n} \cdot z_{2n} = G_n \cdot \sin \psi_n - m_{1n} \cdot g \quad (2)$$

$$m_{2n} \cdot \ddot{z}_{2n} + (d_{1n} + d_{2n}) \cdot \dot{z}_{2n} + (k_{1n} + k_{2n}) \cdot z_{2n} - d_{1n} \cdot \dot{z}_{1n} - k_{1n} \cdot z_{1n} = -m_{2n} \cdot g + d_{2n} \dot{w}_n + k_{2n} w_n$$

By combining Eq. (1) and Eq. (2), the differential equations governing the behavior of the girder element under the distributed load $p(x, z, t)$ can be reformulated as follows:

$$EJ_d \cdot \left(\frac{\partial^4 w}{\partial x^4} + \theta \cdot \frac{\partial^5 w}{\partial x^4 \cdot \partial t} \right) + \rho F_d \cdot \frac{\partial^2 w}{\partial t^2} + \beta \cdot \frac{\partial w}{\partial t} = p(x, z, t) \quad (3)$$

$$p(x, z, t) = \sum_{i=1}^N \xi_i(t) \left[G_n \sin \psi_n - (m_{1n} + m_{2n}) \cdot g \right] \delta(x - a_n)$$

$$m_{1n} \cdot \ddot{z}_{1n} + d_{1n} \cdot \dot{z}_{1n} + k_{1n} \cdot z_{1n} - d_{1n} \cdot \dot{z}_{2n} - k_{1n} \cdot z_{2n} = G_n \cdot \sin \psi_n - m_{1n} \cdot g$$

$$m_{2n} \cdot \ddot{z}_{2n} + (d_{1n} + d_{2n}) \cdot \dot{z}_{2n} + (k_{1n} + k_{2n}) \cdot z_{2n} - d_{1n} \cdot \dot{z}_{1n} - k_{1n} \cdot z_{1n} = -m_{2n} \cdot g + d_{2n} \dot{w}_n + k_{2n} w_n$$

where $\delta(x - a_n)$ is the Dirac delta function, $n=1$ to N (N is number of axles, $N=3$).

The Galerkin method and Green's theorem are applied to Eqs. (3) to transform the system into matrix form, enabling the differential equations governing both the girder element and the vehicle to be expressed as follows:

$$M_e \cdot \ddot{q} + C_e \cdot \dot{q} + K_e \cdot q = f_e \quad (4)$$

In this formulation, M_e , C_e and K_e represent the mass matrix, damping matrix, and stiffness matrix of the system, which includes both the girder and the vehicle, respectively:

$$M_e = \begin{bmatrix} M_{ww} & M_{wz1} & M_{wz2} \\ 0 & M_{z1z1} & 0 \\ 0 & 0 & M_{z2z2} \end{bmatrix}; \quad C_e = \begin{bmatrix} C_{ww} & 0 & 0 \\ 0 & C_{z1z1} & C_{z1z2} \\ C_{z2w} & C_{z2z1} & C_{z2z2} \end{bmatrix}$$

$$K_e = \begin{bmatrix} K_{ww} & 0 & 0 \\ 0 & K_{z1z1} & K_{z1z2} \\ K_{z2w} & K_{z2z1} & K_{z2z2} \end{bmatrix} \quad (5)$$

In this system, the following vectors represent the dynamic response of the combined girder and vehicle system: \ddot{q} is the complex acceleration vector, \dot{q} is complex velocity vector, q is complex displacement vector, f_e is complex forces vector of the system including girder and vehicle;

$$\{\ddot{q}\} = \begin{Bmatrix} \ddot{W} \\ \ddot{Z}_1 \\ \ddot{Z}_2 \end{Bmatrix}; \quad \{\dot{q}\} = \begin{Bmatrix} \dot{W} \\ \dot{Z}_1 \\ \dot{Z}_2 \end{Bmatrix}; \quad \{q\} = \begin{Bmatrix} W \\ Z_1 \\ Z_2 \end{Bmatrix}; \quad \{f_e\} = \begin{Bmatrix} F_w \\ F_{z1} \\ F_{z2} \end{Bmatrix} \quad (6)$$

$W = [u_1 \ \varphi_1 \ u_2 \ \varphi_2]^T$ is the node displacement vector of the beam element in the local coordinate system. The components of this vector are as follows: u_1, φ_1 are the vertical displacement and rotation displacement at the left node; u_2, φ_2 are the vertical displacement and rotation displacement at the right node of the beam element.

In this formulation, M_{ww} represents the mass matrix, C_{ww} denotes the resistance (or damping) matrix, and K_{ww} is the stiffness matrix. These matrices correspond to the basic beam element subjected to both axial and bending forces. The remaining matrices and vectors involved in the system can be found in [22].

$$F_w = \sum_{i=1}^N [G_n \sin \Psi_n - (m_{1n} + m_{2n}) \cdot g] \cdot P_n \quad (7)$$

$$F_{z1} = \begin{Bmatrix} G_1 \sin \Psi_1 - m_{11} \cdot g \\ \vdots \\ G_n \sin \Psi_n - m_{1n} \cdot g \\ \vdots \\ G_N \sin \Psi_N - m_{1N} \cdot g \end{Bmatrix}; \quad F_{z2} = \begin{Bmatrix} -m_{21} \cdot g \\ \vdots \\ -m_{2n} \cdot g \\ \vdots \\ -m_{2N} \cdot g \end{Bmatrix} \quad (8)$$

This study focuses on the scenario where the applied load is a random variable derived from vehicle weighing station data. Consequently, the quantity $m_{1n} = m_{axle\ n} - m_{2n}$ in the Eq (2), (3), (7), and (8) is treated as a random variable, while all other factors are considered deterministic.

4. Result analysis vibration of Song Phan bridge

4.1. Analytical bridge and vehicle model

The Song Phan Bridge is a three-span I-girder structure featuring a link slab, with each span measuring 24 meters in length. The deck slab is constructed from continuously reinforced concrete and includes a 1.2-meter-long flexible joint. The bridge's cross-section comprises 11 prestressed reinforced concrete I-girders, as illustrated in Figure 8.

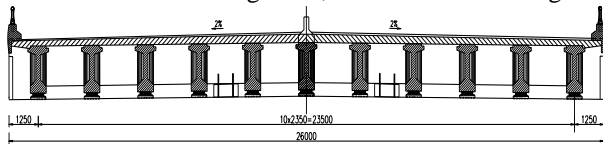


Figure 8. Cross section of Song Phan bridge

The properties of the slab beam are obtained from the design documents of the bridge management unit.

The concrete properties of the slab beam are as follows: Length $L=24$ m, Young's modulus of elasticity $E=3230769.23$ T/m², density $\rho=2.5$ T/m³, friction coefficient $\theta=0.027$ and $\beta=0.01$, the cross sectional area $A=0.9522$ m², second moment of area $I=0.30921$ m⁴. The Link slab (deck): height $h=0.02$ m, the cross sectional area $A=0.47$ m², and second moment of area $I=0.001567$ m⁴.

The bridge structure was discretized using the finite

element method, as illustrated in Figure 9.

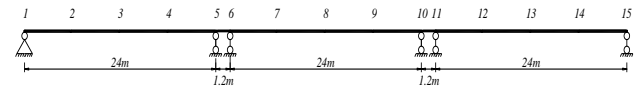


Figure 9. Diagram of a discrete bridge structure

The vehicle parameters, represented by the values m_{11}, m_{12} and m_{13} are random quantities. The remaining data are based on typical vehicles in the region, specifically modeled after a Foton truck with three axles. The parameters for the dumper truck used in the vibration analysis are provided in Table 1.

Table 1. The typical properties of a Foton vehicle

Symbol	Unit	Value
m_{21}	T	0.26
m_{22}, m_{23}	T	0.87
k_{1n}	T/m	120
	T/m	260
k_{2n}	T/m	240
	T/m	380
d_{1n}	Ts/m	0.7344
	Ts/m	0.3672
d_{2n}	Ts/m	0.4
	Ts/m	0.8

4.2. Dau Giay weighing station

After performing the analysis of the random input data set with a sample size of $N=10000$, the obtained results of the DIF vertical displacement at the nodes corresponding to the $\frac{1}{4}$ and $\frac{1}{2}$ positions between the spans are shown in Figures 10 - 13.

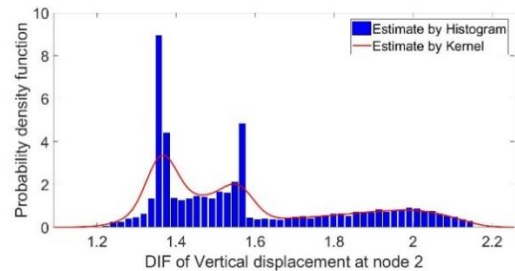


Figure 10. Probability distribution of DIF at node 2

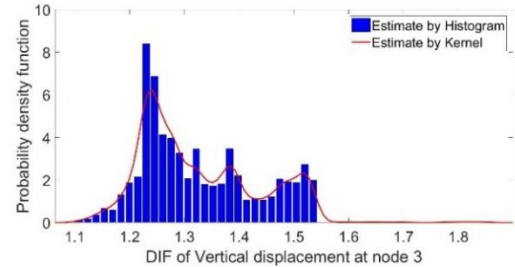


Figure 11. Probability distribution of DIF at node 3

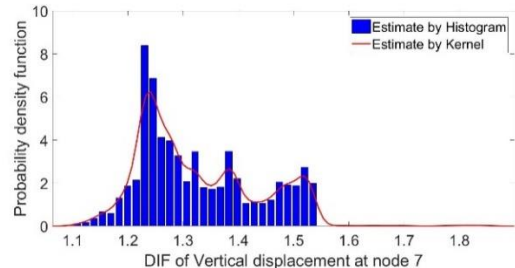


Figure 12. Probability distribution of DIF at node 7

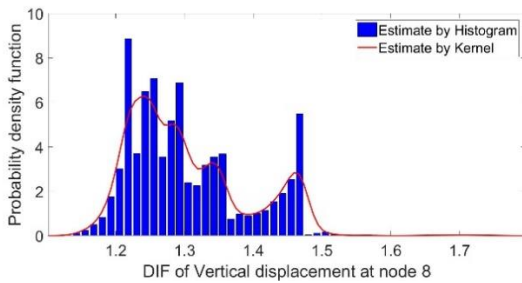


Figure 13. Probability distribution of DIF at node 8

The statistical data regarding the DIF of vertical displacements at the nodes are presented in Table 2.

Table 2. Statistical of DIF at Dau Giay

Statistical	Node 2	Node 3	Node 7	Node 8
Mean	1.466	1.461	1.274	1.272
Max	1.562	1.546	1.318	1.328
Min	1.370	1.216	1.229	1.216
Standard deviation	0.135	0.079	0.063	0.079

4.3. Road Management Area IV

The DIF results of vertical displacement at the nodes corresponding to the $\frac{1}{4}$ and $\frac{1}{2}$ span positions when the bridge is subjected to the load at Road Management Area IV are shown in Figures 14-17.

The statistical data concerning the DIF of vertical displacement at the nodes are presented in Table 3.

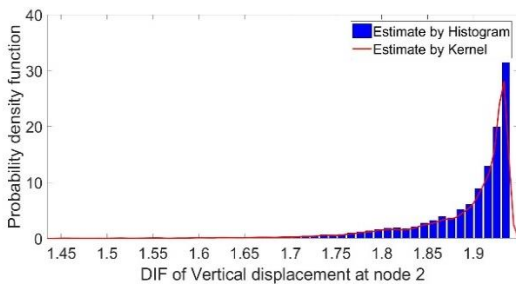


Figure 14. Probability distribution of DIF at node 2

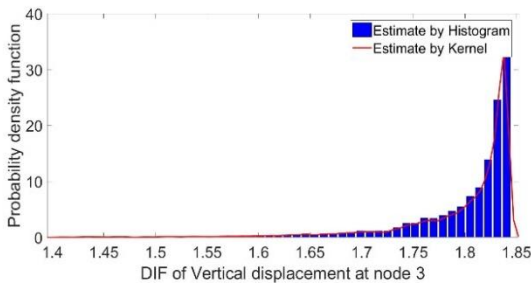


Figure 15. Probability distribution of DIF at node 3

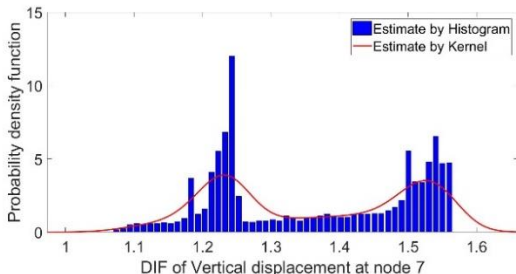


Figure 16. Probability distribution of DIF at node 7

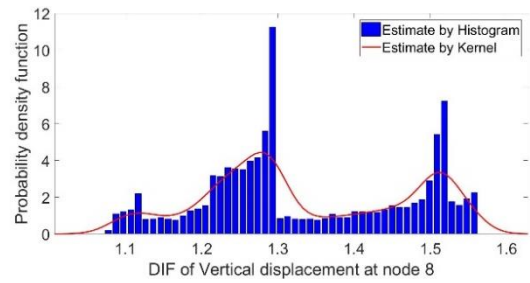


Figure 17. Probability distribution of DIF at node 8

Table 3. Statistical of DIF at RMA IV

Statistical	Node 2	Node 3	Node 7	Node 8
Mean	1.692	1.625	1.317	1.310
Max	1.936	1.546	1.560	0.817
Min	1.448	1.216	1.074	0.821
Standard deviation	0.145	0.079	0.145	0.817

4.4. Compare the results in two corresponding load cases

After analyzing the random input data set and applying it to a three-span girder bridge with a link slab, the results of the vertical displacement analysis were obtained for the loads collected from the Dau Giay weighing station and Road Management Area IV at the nodes.

Table 4. Statistical results of DIF in two cases

Mean value	Node 2	Node 3	Node 7	Node 8
RMA	1.692	1.625	1.317	1.310
DG	1.466	1.461	1.274	1.272
Disparity (%)	13.36	10.09	3.26	2.90

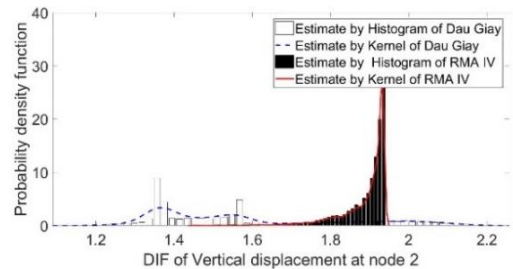


Figure 18. Probability distribution of DIF at node 2

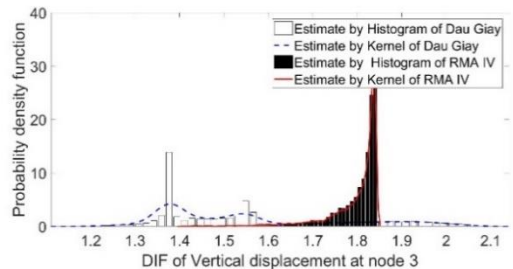


Figure 19. Probability distribution of DIF at node 3

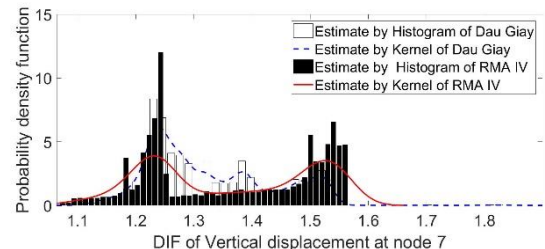


Figure 20. Probability distribution of DIF at node 7

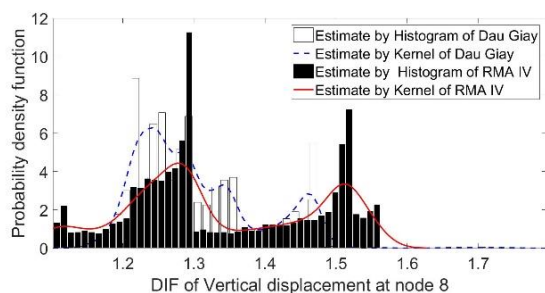


Figure 21. Probability distribution of DIF at node 8

The results of the analysis of the displacement under the action of vehicles with randomly changing loads based on a set of data collected at Road Management Area IV and Dau Giay weighing station show that: the displacement of vertical displacement when analyzed according to vehicle load of Road Management Area IV is always larger than the displacement of displacement when analyzed according to vehicle load of Dau Giay weighing station.

The displacement in the area corresponding to the overload load of Road Management Area IV tends to increase and is larger than the displacement currently applied in bridge design standards according to AASHTO [23] and TCVN 11823-2017 [24], with a value of 1.33.

According to the load distribution and DIF charts, it can be concluded that the DIF increases significantly when the load increases. The increase in DIF and load will be very dangerous for the bridge structure during operation. Accordingly, the increase in dynamic coefficient with a larger load in Road Management Area IV compared to the Dau Giay weighing station needs to be doubly warned to avoid causing danger to the bridge structure.

The analysis results also show that the probability distribution of the DIF random variable is very complicated. The rules of the input random variables (vehicle load) and the output random variables (DIF) are not the same as the probability rules, proving that the mapping relationship between them is not monotonous.

5. Conclusion

This paper examines the DIF of a girder bridge with a link slab, specifically under the influence of vehicles with randomly varying loads. Actual vehicle load data collected and processed at the Dau Giay weighing station and Road Management Area IV were utilized in the analysis. The findings reveal that the probability distribution of the random variable DIF exhibits significant complexity, reflecting the dynamic behavior of the bridge under fluctuating loading conditions. The DIF of displacement analyzed according to the vehicle load of Road Management Area IV is mostly larger than the DIF of displacement when analyzed according to the vehicle load of the Dau Giay weighing station. The DIF in the area corresponding to the overload load of Road Management Area IV tends to increase and is larger than the DIF currently applied in the bridge design standard with a value of 1.33. The increase in dynamic coefficient along with the larger load in Road Management Area IV compared to Dau Giay weighing station needs to be doubly warned to avoid causing danger to the bridge structure.

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