A model for permeability estimation in porous media using a capillary bundle model with the similarly skewed pore size distribution

Nguyen Van Nghia[1], Dao Tan Quy[2] and Luong Duy Thanh[1*]

Abstract: Permeability estimation has a wide range of applications in different areas such as water resources, oil and gas production or contaminant transfer predictions. Few models have been proposed in the literature using different techniques to estimate the permeability from properties of the porous media, such as porosity, grain size or pore size. In this study, we develop a model for permeability for porous media using an upscaling technique. For this, we conceptualize a porous medium as a bundle of capillary tubes with the similarly skewed pore size distribution. The proposed model is related to microstructural properties such as maximum radius, porosity, tortuosity and a characteristic constant of porous media. The model is successfully compared to published experimental data as well as to an existing model in the literature.

Keywords: Permeability, porous media, capillaries, pore size distribution.

1. Introduction

Permeability that defines how easily a fluid flows through porous media is one of the key parameters for modeling flow and transport in saturated porous media. It was shown that the permeability depends on properties of porous media such as porosity, cementation, pore size, pore size distribution (PSD), pore shape and pore connectivity. So far, there have been different techniques in the literature for permeability estimation such as a bundle of capillary tubes (e.g., Nghia et al., 2021), effective-medium approximations (Doyen, 1988), critical path analysis (e.g., Daigle, 2016; Ghanbarian, 2020a). Besides, numerical approaches such as the finite-element, lattice Boltzmann, or pore-network modeling have been also used for the permeability estimation (e.g., Bryant and Blunt, 1992; De Vries et al., 2017). Recently, Nghia et al., 2021 successfully applied a capillary bundle model for porous media whose pores are assumed to follow the fractal power law to predict permeability of porous media under saturated and partially saturated conditions. In addition to the fractal PSD used by Nghia et al., 2021, there have been also other PSDs proposed for porous media in literature. For example, the similarly skewed PSD was used to obtain the streaming potential coupling coefficient in porous media (e.g., Jackson, 2008). The lognormal PSD has been also applied to obtain the relative permeability (e.g., Ghanbarian, 2020b) and the dynamic streaming potential coupling coefficient (e.g., Thanh et al., 2022). Vinogradov et al., 2021 used the non-monotonic PSD that was determined from direct measurements for Berea sandstone samples, thus providing a more realistic description of porous rocks, to simulate the streaming potential coupling coefficient in porous media. To the best of our knowledge, permeability estimation using the similarly skewed PSD, for example, is still lacking in the specific literature.

In this work, we follow the similar approach used by Nghia et al., 2021 to develop a model for permeability under saturated conditions

[1]Faculty of Electrical and Electronics Engineering, Thuyloi University
[2]Faculty of Computer Science and Engineering, Thuyloi University
* Corresponding author
Received 4th Jul. 2022
Accepted 27th Jul. 2022
Available online 31st Dec. 2022

using a simple bundle of capillary tubes model with the similarly skewed PSD. We remark that a capillary bundle model may not be a good representation of the real pore space of geologic porous media. However, it has been proven to be a highly effective tool for description of transport phenomena in porous media (Dullien et al., 1992; Jackson, 2008; Soldi et al., 2017; Nghia A et al., 2021, Vinogradov et al., 2021, Thanh et al., 2022). The proposed model is related to microstructural properties of porous media such as porosity, tortuosity, maximum pore radius and a characteristic parameter of the PSD. Finally, we validate the model by comparing to experimental data and a widely used model available in the literature.

2. Model development

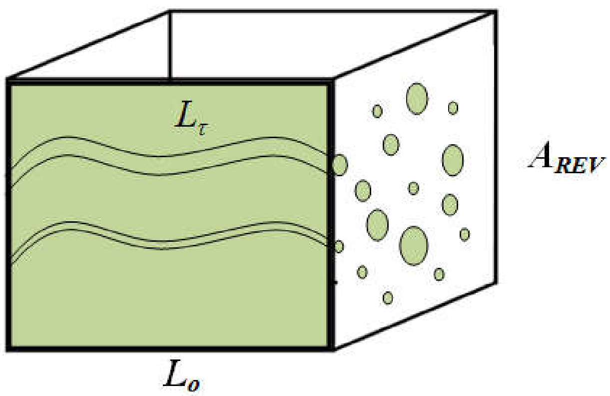

Figure 1. The bundle of capillary tubes model

In order to obtain a model for permeability, we consider a cubic representative elementary volume (REV) of a porous medium of side-length L_o and cross-section area A_{REV} as shown in Fig. 1. In the context of the capillary bundle model, the REV is simply conceptualized as a bundle of tortuous cylindrical capillaries with radii varying from a minimum pore radius $r_{\min}$ to a maximum pore radius $r_{\max}$. All capillaries are parallel and there are no intersections between them (see Fig. 1). The pore size distribution $f(r)$ in the REV is such that the number of capillaries with radius in the range from r to $r + \mathrm{d}r$ is given by $f(r)\mathrm{d}r$. Note that this simple representation of the pore space is based on similar concepts as the classic model of (Kozeny, 1927), which is broadly used in soils. In this context, the total number of capillaries in the REV is determined as

$$N = \int_{r_{\min}}^{r_{\max}} f(r)dr. \tag{1}$$

The similarly skewed PSD for $f(r)$ is given by (e.g., Jackson, 2008; Vinogradov et al. 2021)

$$f(r) = A\left(\frac{r - r_{\max}}{r_{\min} - r_{\max}}\right)^c, \tag{2}$$

where A and c are constants depending on characteristics of porous media. For $c = 0$, the capillary tubes are evenly distributed between $r_{\min}$ and $r_{\max}$. When c increases, the distribution becomes skewed towards smaller capillary radii (e.g., Jackson, 2008).

In the framework of a bundle of capillary tubes, the permeability of the REV is determined by (e.g., Jackson, 2008; Vinogradov et al., 2021)

$$k = \frac{\phi}{8\tau^2}\frac{\int_{r_{\min}}^{r_{\max}} r^4 f(r)dr}{\int_{r_{\min}}^{r_{\max}} r^2 f(r)dr}, \tag{3}$$

where $\prod$ (unitless) and τ (unitless) are porosity and tortuosity of porous media, respectively. Note that the tortuosity is defined as $\tau = L_\tau / L_0$ where L_0 and L_τ are the length of the REV and the length of capillaries as shown in Fig. 1, respectively.

Combining Eq. (2) and Eq. (3), the permeability is approximately obtained as the follows:

$$k = \frac{\phi}{8\tau^2}\frac{12r_{\max}^2}{(c+4)(c+5)}. \tag{4}$$

We remark that $r_{\max}$ is normally much larger than $r_{\min}$ for most of geological porous media (e.g., Liang et al., 2015; Soldi et al., 2017;

Vinogradov et al., 2021). Therefore, we have safely neglected the terms containing r_{min}/ r_{max} during the derivation to obtain Eq. (4) from Eq. (3) and this will be verified in the next section. Eq. (4) is the main contribution of this work. It shows that permeability depends on properties of porous media such as porosity Π, tortuosity τ, maximum radius r_{max} and a characteristic parameter c.

If the PSD of porous media is not available, one can estimate r_{max} from the mean grain diameter d and porosity Π for nonconsolidated granular media using the following (e.g., Liang et al. 2015)

$$r_{max} = \frac{d}{4}\left(\sqrt{\frac{\phi}{1-\phi}} + \sqrt{\frac{1}{1-\phi}}\right) \quad (5)$$

The tortuosity can be estimated from porosity using the following relation for granular media (e.g., Du Plessis and Masliyah, 1991)

$$\tau = \frac{\phi}{1-(1-\phi)^{2/3}} \quad (6)$$

3. Results and discussion

3.1. Sensitivity analysis of the model

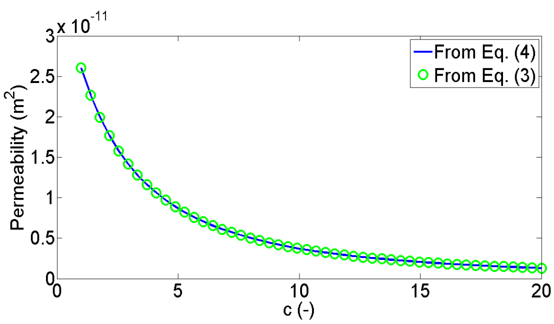

Figure 2. Variation of the permeability with c estimated from the analytical expression - Eq. 4 (the solid line) and from the numerical solution - Eq. 3 (the circles). Input representative parameters are r_{min} = 0.5 µm; r_{max} = 50 µm; ∏ = 0.4 and τ = 1.38.

Figure 2 shows the variation of the permeability with constant c estimated from the analytical expression - Eq. 4 (the solid line) and from the exact expression - Eq. 3 that is numerically solved (the circles) with representative parameters: r_{min} = 0.5 µm; r_{max} = 50 µm; Π = 0.4 and τ = 1.38 that is estimated from Eq. (6) with the knowledge of Π. It is clearly seen that the result obtained from the analytical expression is in very good agreement with that from the exact expression. Therefore, the analytical expression, Eq. 4, is safely used for the permeability estimation. Additionally, one can see that the permeability is sensitive to c and decreases with an increase of c. The reason is that when c increases, there are a larger number of small capillaries in porous media due to the characteristic of the similarly skewed PSD (e.g., Jackson, 2008). Consequently, the ability of water to pass through small capillaries of porous media decreases, leading a decrease of permeability.

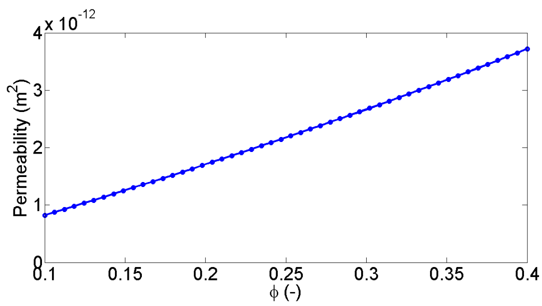

Figure 3. Variation of the permeability with porosity ∏ estimated from Eq. 4. Representative parameters are r_{max} = 50 µm; c = 10 and τ is estimated from Eq. (6) with the knowledge of ∏.

The variation of the permeability k with porosity ∏ is predicted from Eq. (4) in combination with Eq. (6) using representative parameters r_{max} = 50 µm and c = 10 (see Fig. 3). It is seen that k is sensitive with ∏ and increases with increasing ∏ as indicated in the literature (e.g., Kozeny, 1927; Revil and Cathles, 1999).

3.2. Comparison with published data

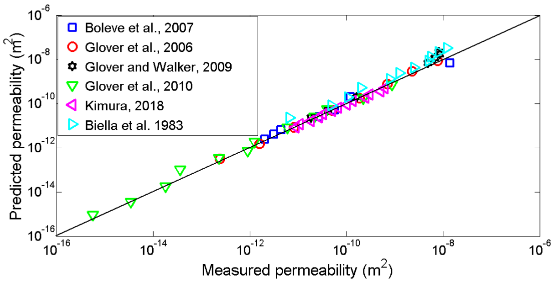

Figure 4. Comparison between estimated permeability from the proposed model - Eq. (4) and 58 experimental data points available in the literature. The solid line is the 1:1 line.

From Eq. (4), we can estimate permeability of porous media if r_{max}, $\prod$, τ and c are known. For example, Fig. 4 shows the comparison between estimated permeability from the proposed model - Eq. (4) and 58 experimental data points available in the literature for uniform grain packs. Namely, we use seven experimental data points reported by Bolève et al., 2007; eight data points reported by Glover et al., 2006; seven data points reported by Glover and Walker, 2009; 12 data points reported by Glover and Dery, 2010; 13 data points reported by Kimura, 2018 and 11 data points reported by Biella et al., 1983. The properties of those samples are reported in the corresponding articles and re-shown in Table 1. Note that r_{max} and τ are estimated from Eq. (5) and Eq. (6), respectively with the knowledge of the grain diameter d and porosity $\prod$ (see Table 1 for each sample). We determine the constant c by seeking a minimum value of the root-mean-square error (RMSE) through the "fminsearch" function in the MATLAB and find c = 6 for all samples. The results in Fig. 4 show that the model prediction is in very good agreement with experimental data reported in the literature.

Table 1. Properties of the glass bead and sand packs

Pack	d (μm)	$\prod$ (unitless)	k (in 10^{-12} m^2)	Reference
Glass bead	56	0.4	2.0	Bolève et al., 2007
	72	0.4	3.1	
	93	0.4	4.4	
	181	0.4	27	
	256	0.4	56	
	512	0.4	120	
	3000	0.4	14000	
Glass bead	20	0.4009	0.24	Glover et al., 2006
	45	0.3909	1.6	
	106	0.3937	8.1	
	250	0.3982	50.5	
	500	0.3812	186.8	
	1000	0.3954	709.9	
	2000	0.3856	2277.3	
	3350	0.3965	7706.9	
Glass bead	3000	0.398	4892	Glover and Walker, 2009
	4000	0.385	6706	
	5000	0.376	8584	
	6000	0.357	8262	
	256	0.399	41.2	

Pack	d (μm)	∏ (unitless)	k (in 10^{-12} m^2)	Reference
	512	0.389	164	
	181	0.382	18.6	
Glass bead	1.05	0.411	0.00057	Glover and Dery, 2010
	2.11	0.398	0.00345	
	5.01	0.380	0.0181	
	11.2	0.401	0.0361	
	21.5	0.383	0.228	
	31	0.392	0.895	
	47.5	0.403	1.258	
	104	0.394	6.028	
	181	0.396	21.53	
	252	0.414	40.19	
	494	0.379	224	
	990	0.385	866.7	
Glass bead	115	0.366	8.8	Kimura, 2018
	136	0.364	10.7	
	162	0.363	18.3	
	193	0.364	26.7	
	229	0.362	33.0	
	273	0.358	51.0	
	324	0.358	67.4	
	386	0.356	102.1	
	459	0.358	134.3	
	545	0.36	246.2	
	648	0.358	299	
	771	0.357	510.4	
	917	0.356	611.9	
Sand	150	0.45	6.7	Biella et al., 1983
	300	0.43	49.2	
	500	0.40	107.7	
	800	0.41	205.1	
	1300	0.40	810.2	
	1800	0.39	1261.4	
	2575	0.37	2563.8	
	3575	0.38	5127.6	
	4500	0.37	5640.4	
	5650	0.37	8204.2	
	7150	0.37	12306.3	

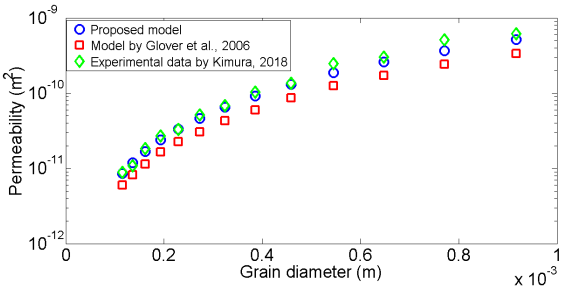

Figure 5. Variation of permeability with grain diameter predicted from the proposed model and the one proposed by Glover et al., 2006 for a set of experimental data by Kimura, 2018.

As previously mentioned, there have been few models available in the literature using different approaches for the permeability estimation (e.g., Kozeny, 1927; Revil and Cathles, 1999; Glover et al., 2006; Ghanbarian, 2020). For example, Glover et al., 2006 proposed a model for the permeability as following:

$$k = \frac{d^2 \phi^{3m}}{4am^2} , \qquad (7)$$

where m and a are parameters taken as 1.5 and 8/3 for the samples that are made up of uniform grains corresponding to the samples in Table 1.

Figure 5 shows the comparison between the proposed model given by Eq. (4) and the one given by Glover et al., 2006 for a representative set of data reported by Kimura, 2018, for example (see Table 1). The RMSE values for the proposed model and the model by Glover et al., 2006 are found to be 4.2×10^{-11} m^2 and 7.8×10^{-11} m^2, respectively. It is seen that the proposed model can provide a slightly better estimation than Glover et al., 2006 with a suitable constant c that is earlier found to be 6 for uniform glass bead and sand packs.

4. Conclusion

We present a model for the permeability estimation in porous media under saturated conditions using a bundle of capillary tubes model with the similarly skewed PSD and an upscaling technique. The proposed model is expressed in terms of properties of porous media (maximum radius, porosity, tortuosity and a characteristic constant c). The model is successfully validated by comparisons with 58 samples of uniform glass bead and sand packs reported in the literature and with an existing model proposed by Glover et al., 2006. Along with other models in the literature, the analytical model developed in this work opens up many possibilities for investigation of fluid flow in porous media.

Acknowledgements

This research is funded by Thuyloi University Foundation for Science and Technology under grant number TLU.STF.21-06.

References

Biella G, Lozej A and Tabacco I (1983), *"Experimental study of some hydrogeophysical properties of unconsolidated porous media"*, Groundwater, 21, 741-751.

Bolève A, Crespy A, Revil A, Janod F and Mattiuzzo J. L (2007), *"Streaming potentials of granular media: Inuence of the dukhin and reynolds numbers"*, J. Geophys. Res.: Solid Earth, 112 (B8), 1-14.

Bryant S and Blunt M (1992), *"Prediction of relative permeability in simple porous media"* Phys. Rev. A, 46 (4), 2004-2011.

Daigle H (2016), *"Application of critical path analysis for permeability prediction in natural porous media"*, Advances in Water Resources, 96, 43-54.

De Vries E, Raoof A and Genuchten M (2017), *"Multiscale modelling of dual-porosity porous media; a computational pore-scale study for flow and solute transport"*, Advances in Water Resources, 105, 82-95.

Doyen P. M, (1988), *"Permeability, conductivity, and pore geometry of sandstone"*, J. Geophys. Res.: Solid Earth, 93, 7729-7740.

Dullien F. A. L (1992), *"Porous media: Fluid transport and pore structure"*, Academic Press, San Diego.

Du Plessis J. P and Masliyah J. H (1991), *"Flow through isotropic granular porous media"*, Transp. Porous Media, 6, 207–221.

Ghanbarian B (2020a), *"Applications of critical path analysis to uniform grain packings with narrow conductance distributions: I. single-phase permeability"*, Advances in Water Resources, 137, 103529.

Ghanbarian B (2020b), *"Applications of critical path analysis to uniform grain packings with narrow conductance distributions: II. water relative permeability"*, Advances in Water Resources, 137, 103524.

Glover P, Zadjali I. I and Frew K. A (2006), *"Permeability prediction from micp and nmr data using an electrokinetic approach"*, Geophysics, 71, 49-60.

Glover P. W. J and Dery N (2010), *"Streaming potential coupling coefficient of quartz glass bead packs: Dependence on grain diameter, pore size, and pore throat radius"*, Geophysics, 75, 225-241.

Glover P. W. J and Walker E (2009), *"Grain-size to effective pore-size transformation derived from electrokinetic theory"*, Geophysics, 74(1), 17-29.

Jackson M. D (2008), *"Characterization of multiphase electrokinetic coupling using a bundle of capillary tubes model"*, J. Geophys. Res.: Solid Earth, **113** (B4), 005490.

Kimura M (2018), *"Prediction of tortuosity, permeability, and pore radius of water-saturated unconsolidated glass beads and sands"*, The Journal of the Acoustical Society of America, 141, 3154-3168.

Kozeny J (1927), *"Uber kapillare leitung des wassers im boden aufsteigversikeung und anwendung auf die bemasserung"* Math-Naturwissen-schaften, 136, 271-306.

Liang M, Yang S, Miao T and Yu B (2015), *"Analysis of electroosmotic characters in fractal porous media"*, Chemical Engineering Science, 127.

Nghia A N. V, Jougnot D, Thanh L. D, Van Do P, Thuy T. T. C, Hue D. T. M, Nga P. T. T (2021), *"Predicting water flow in fully and partially saturated porous media, a new fractal based permeability model"*, Hydrogeology Journal, 29, 2017–2031.

Nghia N. V, Hung N. M, Thanh L. D (2021), *"A model for electrical conductivity of porous materials under saturated conditions"*. VNU J. Sci.: Mathematics - Physics, 37(2), 13-21.

Revil A and Cathles L. M. (1999), *"Permeability of shaly sands"*, Water Resources Research, 3, 651-662.

Soldi M, Guarracino L and Jougnot D (2017), *"A simple hysteretic constitutive model for unsaturated flow",* Transport in Porous Media, 120, 271-285.

Thanh L. D, Jougnot D, Solazzi S. G, Nghia, N. V, Van Do P (2022), *"Dynamic streaming potential coupling coefficient in porous media with different pore size distributions"*, Geophys. J. Int., 229, 720–735.

Vinogradov J, Hill R, Jougnot D (2021), *"Influence of pore size distribution on the electrokinetic coupling coefficient in two-phase flow conditions"*, Water, 13, 2316.