OPTICALLY DETECTED ELECTRON - PHONON RESONANCES IN HYPERBOLIC PÖSCHL-TELLER QUANTUM WELLS

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Abstract: The explicit expressions for linearly optical conductivity and absorption power of electromagnetic wave caused by confined electrons in the hyperbolic quantum well is obtained in the case of electron-longitudinal optical phonon scattering. Linearly optically detected electrophonon resonance (ODEPR) effect in a specific GaAs/AlAs hyperbolic quantum well with Pöschl-Teller potential type is investigated. Conditions for ODEPR are discussed based on the curves expressing the dependence of absorption power on the photon energy. From these curves we obtained ODEPR - linewidths as profiles of the curves. Computational results show that linear ODEPR- linewidths increase with temperature and decrease with well parameters.
Keywords: absorption power, quantum well, hyperbolic, Pöschl Teller potential, ODEPR- linewidths

1 INTRODUCTION

In recent years, electron-phonon resonance effect (EPR) and optically detected electron-phonon resonance (ODEPR) is being interested by domestic and international physicists. These studies have contribution in clarifying new properties of electrons under the effect of external fields, therefore provide information of optical properties of semiconductors to the technology of fabricating optoelectronic components.

EPR phenomena arise from an electron scattering due to the absorption and emission of phonons when the energy difference of two electric subbands equals the longitudinal-optical (LO) phonon energy. If these processes are accompanied by the absorption or emission of photons, we will have the ODEPR effect. The EPR effect has been started to study since 1972 by Bryskin and Firsov for the nondegenerate

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semiconductor in strong electric field [1]. The study of EPR effects is very important in understanding transport phenomena in the modern quantum devices. For electrons in quantum wells, the investigation of multi-subband transport effects such as the effective mass, the energy levels, and the electron-phonon interaction has received some attentions [2, 3, 4, 5]. There are now a lot of works focusing on this phenomenon in two-dimensional semiconductor [6, 7, 8], and quantum wires [9, 10]. Most studies, however, focus on conventional quantum wells such as square potential quantum wells, parabolic potential quantum wells, etc. Hyperbolic quantum well with Pöschl-Teller potential type is still new direction with some studies and has attracted some interest in recent years [11, 13, 14, 15]. Tong [12] suggested several applications in semiconductor heterojunction devices and in optical systems. Tong and Kiriushcheva [13] showed that it can be used in the reduction of noise in resonance tunneling devices and other devices. Radovanovic et al. [14] worked several intersubband absorption properties of the potential. Yildirim and Tomak [15] studied some intersubband nonlinear optical properties of the potential.

In this paper, we investigate the linear ODEPR effect and linewidths in the hyperbolic quantum well with Pöschl-eller potential type. First, we derive the analytical expression of linear absorption power. From curves on graphs of absorption power as a function of photon energy, we obtain ODEPR-linewidths as a profiles of curves by using the Profile method presented in one of our previous papers [16]. The dependence of linear linewidths on temperature and the well parameter are discussed. The paper is organized as follows: The model and theoretical framework is described in Section 2, the results and discussions are presented in Section 3, and finally, the conclusions are given in Section 4.

2 MODEL AND THEORY

We consider a quantum well with the electron confined potential expressed in the form of [17]

$$V(z) = \frac{V_1 - V_2 \cosh(\alpha z)}{\sinh^2(\alpha z)}, \tag{1}$$

where V_1, V_2 and α are three parameters representing the properties of potential.

Solving Schrodinger equation, we obtain the energy spectrum and corresponding electron wave function as follows [18]:

$$E_\alpha = = E_{n_z} + E_{k_x k_y} = \frac{\hbar^2}{2m^*}\left(k_x^2 + k_y^2\right) - \frac{\alpha^2}{8}[\upsilon - \mu - (1+2n)]^2, \tag{2}$$

$$n = 0, 1, 2, ..., \left[\frac{1}{2}\left(\upsilon - \mu - 1\right)\right].$$

$$\psi_\alpha(x,y,z) = \frac{1}{\sqrt{L_x L_y}} e^{i(k_x x + k_y y)} C u^\delta (1-u)^\varepsilon {}_2F_1\left[-n, n+2(\delta+\varepsilon+1/4); 2\delta+1/2; u\right], \tag{3}$$

where $\delta = \frac{1}{4} + \frac{\mu}{2}$, $\mu = \frac{1}{2\alpha}\sqrt{8(V_1+V_2)+\alpha^2}$, $v = \frac{1}{2\alpha}\sqrt{8(V_1-V_2)+\alpha^2}$, $\varepsilon = \frac{\sqrt{-2E}}{\alpha}$, $u = \tanh^2\frac{\alpha z}{2}$. The coefficient C is expressed in the form of Gamma function:

$$C = \sqrt{2\alpha\varepsilon\Gamma(n+\mu+1)\Gamma(n+\mu+2\varepsilon+1)/n!\Gamma(\mu+1)^2\Gamma(n+2\varepsilon+1)}.$$

Consider an electromagnetic wave with angular frequency ω and amplitude E_0, the linear absorption power delivered to the system is given by

$$P_{0z}(\omega) = \frac{E_0^2}{2}\,\mathrm{Re}\left[\sigma_{zz}(\omega)\right], \tag{4}$$

here $\sigma_{zz}(\omega)$ is the z component of optical conductivity tensor:

$$\sigma_{zz}(\omega) = -e\lim_{\Delta\to 0^+}\sum_{\alpha\beta}\langle z\rangle_{\alpha\beta}\langle j_z\rangle_{\beta\alpha}\frac{f_\beta - f_\alpha}{\hbar\bar{\omega} - E_{\beta\alpha} - \Gamma_0^{\alpha\beta}(\bar{\omega})}, \tag{5}$$

where e is the electric charge of the electron, $\langle X\rangle \equiv \langle\alpha|X|\beta\rangle$ is the matrix element of operator X, $f_{\alpha(\beta)}$ is the Fermi-Dirac distribution function of the electron at energy state $E_{\alpha(\beta)}$, $\bar{\omega} = \omega - i\Delta$ $(\Delta \to 0^+)$; $\Gamma_0^{\alpha\beta}(\bar{\omega})$ is called the spectral lineshape function. The matrix element of position operator z is calculated by

$$\begin{aligned}\langle z\rangle_{\alpha\beta} &= \delta_{k_x',k_x}\delta_{k_y',k_y}\int_0^{L_z} CC' u^{2\delta}(1-u)^{2\varepsilon}{}_2F_1\left[-n', n'+2(\delta+\varepsilon+1/4); 2\delta+1/2; u\right] \\ &\times\ {}_2F_1\left[-n, n+2(\delta+\varepsilon+1/4); 2\delta+1/2; u\right] z\,dz = \delta_{k_x',k_x}\delta_{k_y',k_y} I_1.\end{aligned}$$

The matrix element of current density operator j_z is calculated by

$$\begin{aligned}\langle j_z\rangle_{\beta\alpha} &= \frac{ie\hbar}{m_e^*}\delta_{k_x',k_x}\delta_{k_y',k_y}CC'\int_0^\infty u^\delta(1-u)^\varepsilon{}_2F_1\left[-n', n'+2(\delta+\varepsilon+1/4); 2\delta+1/2; u\right] \\ &\times \left[\delta u^{\delta-1}(1-u)^\varepsilon + u^\delta\varepsilon(1-u)^\varepsilon - 1\right].{}_2F_1\left[-n, n+2\left(\delta+\varepsilon+\frac{1}{4}\right); 2\delta+\frac{1}{2}; u\right] \\ &+ u^\delta(1-u)^\varepsilon\frac{(-n)(n+2\delta+2\varepsilon+1/2)}{2\delta+\frac{1}{2}} \\ &\times\ {}_2F_1\left[-n+1, n+1+2\left(\delta+\varepsilon+\frac{1}{4}\right); 2\delta+\frac{3}{2}; u\right]dz = \frac{ie\hbar}{m^*}\delta_{k_x',k_x}\delta_{k_y',k_y}I_2\end{aligned}$$

Take the real part of $\sigma_{zz}(\omega)$, we obtain the general expression of the linear absorption power in z direction

$$P_0(\omega) = \frac{e^2\hbar E_{0z}^2}{2m_e^*}\sum_{k_x,k_y,n}\sum_{k'_x,k'_y,n'}\frac{\left(f_{k'_x,k'_y,n'} - f_{k_x,k_y,n}\right)B_0(\omega)}{(\hbar\omega - E_{\beta\alpha})^2 + B_0^2(\omega)}\delta_{k_x',k_x}\delta_{k_y',k_y}I_1 I_2. \tag{6}$$

In the above equation, we denote $E_{\beta\alpha} = E_\beta - E_\alpha = E_{k'_x,k'_y,n'} - E_{k_x,k_y,n}$. The relaxation rate B_0 gets the form as follows:

$$
\begin{aligned}
B_0(\omega) = {} & \frac{L_x L_y D m_e^*}{16\pi^3\hbar^2(f_\beta - f_\alpha)} \sum_{n''} \Bigg\{ \Bigg(\frac{F_{01}}{M_{01}M_{02}} \left[\frac{1}{(k'_x + M_{01})^2} + \frac{1}{(k'_x - M_{01})^2} \right] \\
& \times \left[\frac{1}{(k'_y + M_{02})^2} + \frac{1}{(k'_y - M_{02})^2} \right] + \frac{F_{02}}{M_{03}M_{04}} \left[\frac{1}{(k'_x + M_{03})^2} + \frac{1}{(k'_x - M_{03})^2} \right] \\
& \times \left[\frac{1}{(k'_y + M_{04})^2} + \frac{1}{(k'_y - M_{04})^2} \right] \Bigg) N_{11} \\
& + \Bigg(\frac{F_{03}}{M_{05}M_{06}} \left[\frac{1}{(-k_x + M_{05})^2} + \frac{1}{(k_x + M_{05})^2} \right] \\
& \times \left[\frac{1}{(-k_y + M_{06})^2} + \frac{1}{(k_y + M_{06})^2} \right] + \frac{F_{04}}{M_{07}M_{08}} \left[\frac{1}{(-k_x + M_{07})^2} + \frac{1}{(k_x + M_{07})^2} \right] \\
& \times \left[\frac{1}{(-k_y + M_{08})^2} + \frac{1}{(k_y + M_{08})^2} \right] \Bigg) N_{31} \Bigg\}. \qquad (7)
\end{aligned}
$$

where $f_\alpha(f_\beta)$ is the Fermi-Dirac distribution function of electron with energy $E_\alpha(E_\beta)$,

$$
\begin{aligned}
D &= \frac{e^2\hbar\omega_{LO}}{2\varepsilon_0}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0}\right); \\
M_{01} = M_{03} &= \left[k_x^2 + \frac{2m_e^*}{\hbar^2}(\hbar\omega \pm \hbar\omega_{LO} - E_{n''} + E_n) \right]^{\frac{1}{2}}; \\
M_{02} = M_{04} &= \left[k_y^2 + \frac{2m_e^*}{\hbar^2}(\hbar\omega \pm \hbar\omega_{LO} - E_{n''} + E_n) \right]^{\frac{1}{2}}; \\
M_{05} = M_{07} &= \left[{k'}_x^2 - \frac{2m_e^*}{\hbar^2}(\hbar\omega \pm \hbar\omega_{LO} - E_{n'} + E_{n''}) \right]^{\frac{1}{2}}; \\
M_{06} = M_{08} &= \left[{k'}_y^2 - \frac{2m_e^*}{\hbar^2}(\hbar\omega \pm \hbar\omega_{LO} - E_{n'} + E_{n''}) \right]^{\frac{1}{2}}; \\
F_{01} &= (1 + N_q)(1 - f_\alpha)\left(1 + \exp\left[\theta\left(\frac{\hbar^2 {M_{01}}^2}{2m_e^*}{M_{02}}^2 + E_{n''} - E_F\right)\right]\right)^{-1}; \\
& \quad - N_q f_\alpha \left[1 - \left(1 + \exp\left[\theta \frac{\hbar^2 {M_{01}}^2}{2m_e^*}{M_{02}}^2 + E_{n''} - E_F\right]\right)^{-1} \right] \\
F_{02} &= N_q(1 - f_\alpha)\left(1 + \exp\left[\theta\left(\frac{\hbar^2 {M_{03}}^2}{2m_e^*}{M_{04}}^2 + E_{n''} - E_F\right)\right]\right)^{-1} \\
& \quad - (1 + N_q) f_\alpha \left[1 - \left(1 + \exp\left[\theta\left(\frac{\hbar^2 {M_{03}}^2}{2m_e^*}{M_{04}}^2 + E_{n''} - E_F\right)\right]\right)^{-1} \right];
\end{aligned}
$$

$$F_{03} = (1+N_q)\, f_\beta \left[1 - \left(1 + \exp\left[\theta\left(\frac{\hbar^2 {M_{05}}^2}{2m_e^*}{M_{06}}^2 + E_{n''} - E_F\right)\right]\right)^{-1}\right]$$
$$- N_q\,(1-f_\beta)\left(1 + \exp\left[\theta\left(\frac{\hbar^2 {M_{05}}^2}{2m_e^*}{M_{06}}^2 + E_{n''} - E_F\right)\right]\right)^{-1};$$
$$F_{04} = N_q f_\beta \left[1 - \left(1 + \exp\left[\theta\left(\frac{\hbar^2 {M_{07}}^2}{2m_e^*}{M_{08}}^2 + E_{n''} - E_F\right)\right]\right)^{-1}\right]$$
$$- (1+N_q)\,(1-f_\beta)\left(1 + \exp\left[\theta\left(\frac{\hbar^2 {M_{07}}^2}{2m_e^*}{M_{08}}^2 + E_{n''} - E_F\right)\right]\right)^{-1};$$
$$N_{11} = N_{21} = C^2 C'^2 \frac{1}{\alpha^2}\left(18 - 30\varepsilon + 12\varepsilon^2\right)^2 \left(\frac{1+2\varepsilon}{2\delta + \frac{1}{2}}\right)^2 \frac{2\pi}{L_z};$$
$$N_{31} = N_{41} C^2 C''^2 \frac{1}{\alpha^2}\left(18 - 30\varepsilon + 12\varepsilon^2\right)^2 \left(\frac{1+2\varepsilon}{2\delta + \frac{1}{2}}\right)^2 \frac{2\pi}{L_z}.$$

Substituting $B_0\,(\omega)$ into Eq. (6), we obtained the explicit expression of linear absorption power in the quantum well. We found that the analytical result is quite complex. However, physical meaning can be obtained from numerical computation and graphical plotting.

3 NUMERICAL RESULTS AND DISCUSSIONS

To clarify the obtained analytical results, we carry out some numerical computations and graphical plotting for a specific AlGaAs/GaAs quantum well. Parameters used are [19, 20]: electrical charge $e = 1.6 \times 10^{-19}$ C, electron effective mass $m^* = 6.097 \times 10^{-32}$ kg, Planck constant $\hbar = 1.0544 \times 10^{-34}$ Js, Boltzmann constant $k_\beta = 1.38066 \times 10^{-23}$ J/K, permittivity of free space $\epsilon_0 = 13.5$, high frequency permeability $\chi_\infty = 10.9$, static frequency permeability $\chi_0 = 12.9$, LO-phonon energy $\hbar\omega_{LO} = 36.25$ meV, external field amplitude $E_0 = 10^5$ V/m.

The expression of the linear ODEPR condition is expressed by

$$\hbar\omega \pm E_{\beta\alpha} \pm \hbar\omega_{LO} = 0. \tag{8}$$

When the linear ODEPR conditions are satisfied, in the course of scattering events, electrons in the state $|\alpha\rangle$ could make transition to state $|\beta\rangle$ by absorbing one photon of energy $\hbar\omega$, accompanied with the absorption and/or emission of a LO-phonon of energy $\hbar\omega_{LO}$.

Figure 1 describes the dependence of the linear absorption power $P_0\,(\omega)$ on the photon energy at $T = 200$ K, with parameter $\alpha = 2.2 \times 10^8\ m^{-1}$. From the figure we can see three resonance peaks describing the different transitions of electrons, satisfying the various resonance conditions:

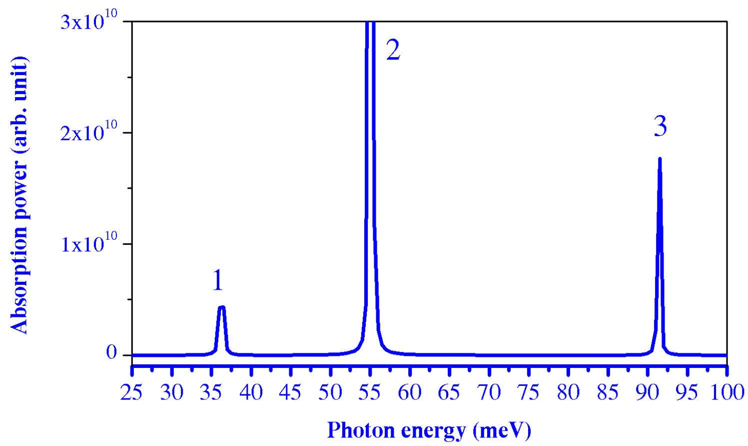

Figure 1: Linear absorption power as a function of photon energy. Here: $T = 200$ K, $\alpha = 2.2 \times 10^8\ m^{-1}$.

+ The first peak at $\hbar\omega = 36.25$ meV satisfies the resonance condition $\hbar\omega = \hbar\omega_{LO}$ corresponding to intrasubband, where one phonon with energy of $\hbar\omega_{LO}$ is emitted.

+ The second peak at $\hbar\omega = 55.1176$ meV satisfying the resonance condition $\hbar\omega = E_\beta - E_\alpha = (67.7013 - 12.5837)$ meV , corresponds to the process in which an electron from the α state absorbs a photon and moves to β state. This process does not include any absorption or emission of phonon.

+ The third peak at $\hbar\omega = 91.3676$ meV satisfies the resonance condition $\hbar\omega = E_{\beta\alpha} + \hbar\omega_{LO} = (55.1176 + 36.25)$ meV or $E_\beta = E_\alpha + \hbar\omega - \hbar\omega_{LO}$. This is the process in which an electron in the state of energy E_α absorbs a phonon and transits to the state of energy E_β. This process is accompanied by emitting a phonon with energy of $\hbar\omega_{LO}$.

Figure 2 shows the dependence of $P_0(\hbar\omega)$ on photon energy with different values of temperature. From the figure we can see that ODEPR peaks locate at the same position ($\hbar\omega$ =91.3676 meV) and the linewidths increase with the temperature as shown in Fig. 3. This result is consistent with the theoretical results of Kang and co-works [21, 22, 23, 24], of Li and of Ning's results [25], and experimental data of Unuma [26]. This can be explained that as temperature increases, the probability of electron-phonon scattering increases, and so do the linewidths.

Figure 4 shows the dependence of $P(\hbar\omega)$ on photon energy at different values of α parameter. From the figure we can see that ODEPR peaks locate at the different

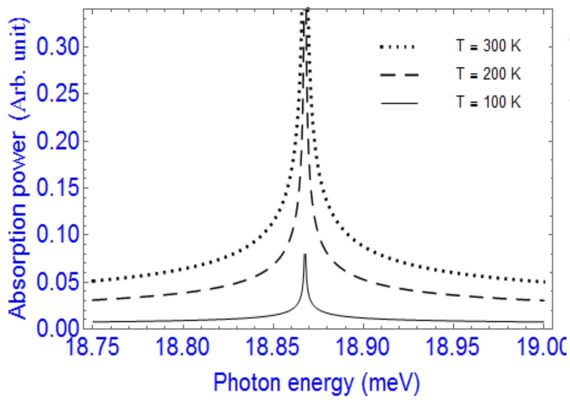

Figure 2: Dependence of $P_0(\hbar\omega)$ on photon energy at different temperature.

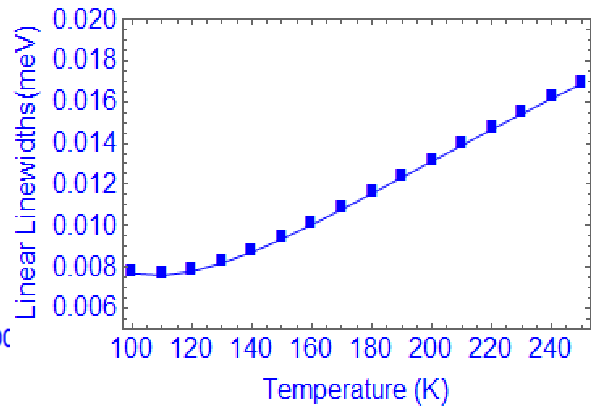

Figure 3: Dependence of linear ODEPR-linewidths on temperature.

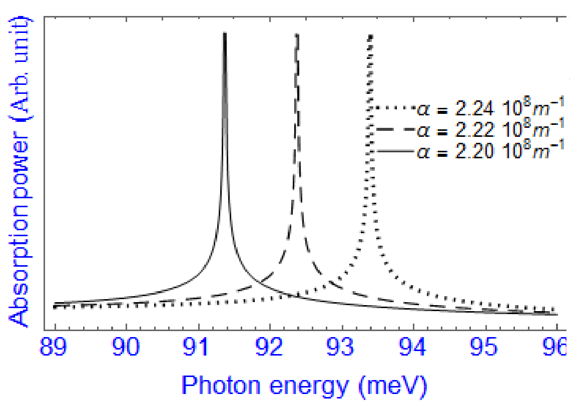

Figure 4: Dependence of $P_0(\hbar\omega)$ on photon energy at different value of α.

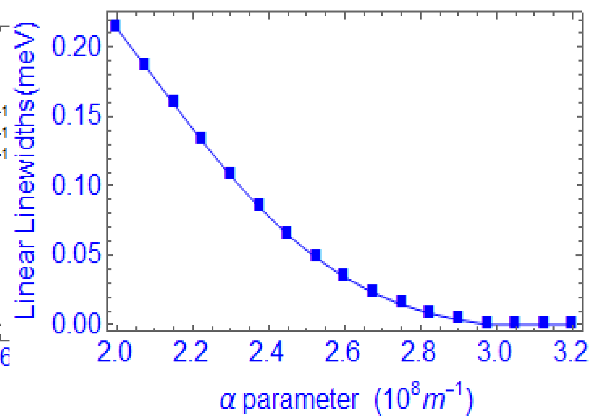

Figure 5: Dependence of linear ODEPR-linewidths on α parameter.

positions with different values of $\hbar\omega$. The reason for this is that the energy spectrum depends on α, that leads to the dependence of resonance peaks on photon energy. From Fig. 4 we can found that linewidths decrease as the parameter α increases as shown in Fig. 5. This can be explained that when the parameter α increases the confinement of electron decreases. This results in the reduction the probability of the electron - phonon scattering, so do the linewidths.

4 CONCLUSIONS

We have studied the ODEPR effect in hyperbolic quantum well with Pöschl - Teller type potential. Special attention is given to the behavior of the ODEPR lineshape, such as the splitting of ODEPR peaks from selection rules. The splitting of peaks on resonant curves satisfied the ODEPR conditions. From the graphs of the linear absorption power $P_0(\omega)$, we obtained ODEPR-linewidths as profiles of curves. Computational results show that the ODEPR-linewidths increase with temperature but decrease with potential parameter. The obtained results are clearly interpreted. It is to be regretted that this result cannot be verified experimentally yet because no experimental results are available at the moment. However, we hope that these our results are helpful in future experiments.

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REFERENCES

[1] V.V.Bryskin, YuA. Firsov (1971), Sov. Phys. JETP **61**, 2373.
[2] S. Briggs, J.F. Leburton (1989), Superlatt. Microstruct. **5**, 145.
[3] A. Kastalsky, F.M. Peeters, W.K. Chan, L.T. Florez, J.P. Harbison (1991), Appl. Phys. Lett. **59**, 1708.
[4] W. Xu, F.M. Peeters, J.T. Devreese (1993), Phys. Rev. B **48**, 1562.
[5] SeGi Yu, V.B. Pevzner, K.W. Kim, M.A. Stroscio (1998), Phys. Rev. B **58**, 3580.
[6] N. L.Kang, and S.D. Choi (2002), J.Phys.: Condens. Matter **14**, 9733.
[7] S. C. Lee (2008), J. Korean Phys. Soc. **52(6)**, 1832.
[8] Y. He, Q. S. Zhu, Z. T. Zhong, G. Z. Zhang, J. Xiao, Z. P. Cao, X. H. Sun and H. Z. Yang (1998), Appl. Phys. Lett.**73**, 1131.
[9] S. C Lee, J. W. Kang, H. S. Ahn, M. Yang, N. L. Kang, Physica E **28** (2005), 402.
[10] SeGi Yu, V. B. Pevzner, and K. W.Kim (1998), Phys. Rev. B **58**, 35803.
[11] N. L., Lee Y. J. and Choi S. D. (2011), Curr. Appl. Phys. **58**, 538.
[12] B. Y. Tong (1997), Solid State Commun. **104**, 679.
[13] B. Y. Tong and N. Kiriushcheva (1997), Phys. Lett. A **229**, 49.
[14] J. Radovanovic, V. Milanavic, Z. Ikonic, and D. Indjin (2000), Phys. Lett. A **269**, 179.
[15] H. Yildirim and M. Tomak (2005), Phys. Rev. B **7**2, 115340.
[16] T.C. Phong, H.V. Phuc (2011), Mod. Phys. Lett. B **25**, 1003.
[17] V.G. Bagrov, D.M. Gitman (1990), Exact Solution of Relativistic Wave Equations, Kluwer Academic Publishers, Dordrecht.
[18] Shi-Hai Dong, A. Gonzalez-Cisneros (2008) Annals of Physics **323**, 1136.
[19] M. Masale, N.C. Constantinou (1993), Phys. Rev. B **48**, 11128.
[20] S.V. Branis, G. Lee, K.K. Bajaj (1993), Phys. Rev. B **47**, 1316.
[21] N.L. Kang, Y.S. Ji, H.J. Lee, S.D. Choi (2003), J. Kor. Phys. Soc. **42**, 379.
[22] N.L. Kang, S.D. Choi (2002), J. Phys.: Condens. Matter **14**, 9733.
[23] N.L. Kang, D.H. Shin, S.N. Yi, S.D. Choi (2005), J. Kor. Phys. Soc. **46**, 1040.

[24] N.L. Kang, Y.J. Lee, S.D. Choi (2004), J. Kor. Phys. Soc. **44**, 1535.

[25] J. Li, C.N. Ning (2004), Phys. Rev. B **70**, 125309.

[26] T. Unuma, T. Takahashi, T. Noda, M. Yoshita, H. Sasaki, M. Baba, H. Akiyama (2001), Appl. Phys. Lett. **78**, 3448.